

SECTION A [30 MARKS]
ANSWER ALL QUESTION

Question 1

[30]

For each question there are FOUR responses: A, B, C and D. Choose the corresponding letter of your response and CIRCLE it neatly. NO score will be awarded, if you circle more than ONE letter.

i. A banker's discount and gain of a bill are Nu 345 and Nu 234 respectively. Find the true discount

- A 79
- B 111
- C 121
- D 579

Criteria	Marks
Circles the correct option	2
Circles more than ONE alternative	0
Circles none of the alternatives	0

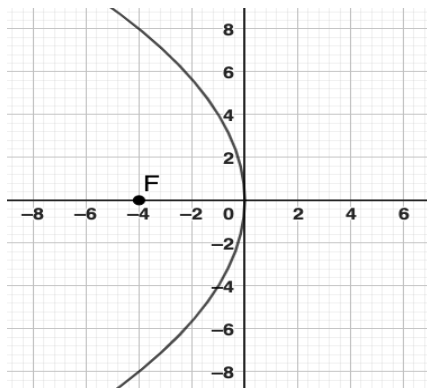
True discount = Bankers' discount – Bankers' gain

$$= \text{Nu } 345 - \text{Nu } 234$$

$$= \text{Nu } 111$$

Ans: B 111

ii. From the graph, find the equation of the directrix.



A $x - 4 = 0$

B $x + 4 = 0$

C $y - 4 = 0$

D $y + 4 = 0$

$y^2 = -16x \Rightarrow y^2 = 4(-4)x$, here $a = -4$

Equation of directrix: $x = a \Rightarrow x = 4 \Rightarrow x - 4 = 0$

Ans: A $x - 4 = 0$

iii. Bhutan Lottery Limited wants to launch a new online game to generate 4-digit serial number with the digits 0, 1, 3, 4, 5, 7, 8. How many 4-digit different even numbers can be formed when repetition of the digits is allowed?

A 588 ways

B 686 ways

C 882 ways

D 1029 ways

The unit's place can be filled up by any of the digit (0, 4, 8) in ${}^3P_1 = 3$

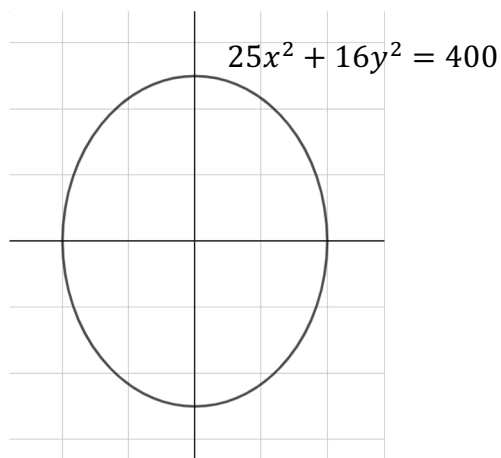
The thousand's place can be filled up by any of the digit (1, 3, 4, 5, 7, 8) in ${}^6P_1 = 6$

The remaining 2 places can be filled up by 7×7 respectively as repetition is allowed.

Therefore, the number of ways ${}^3P_1 \times {}^6P_1 \times 7 \times 7 = 882$ ways

Ans: C 882 ways

iv. Using the given diagram, find the eccentricity.



- A $\frac{\sqrt{41}}{4}$
- B $\frac{\sqrt{41}}{5}$
- C $\frac{3}{5}$
- D $\frac{4}{5}$

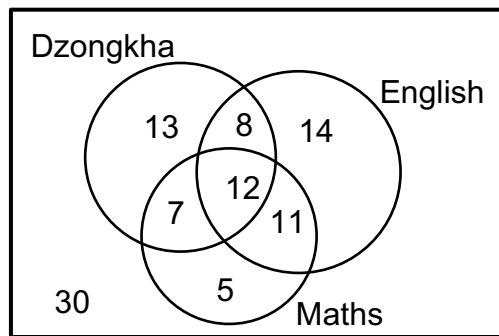
From the given equation $25x^2 + 16y^2 = 400$, we have

$$a^2 = 25 \Rightarrow a = 5 \quad \text{and} \quad b^2 = 16 \Rightarrow b = 4$$

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

Ans: C $\frac{3}{5}$

v. Find the probability that a randomly chosen student studied only one of the subjects.



- A 0.05
- B 0.13
- C 0.14
- D 0.32

Sample space (S) = 100

Favourable outcome $n(A) = \text{English (14)} + \text{Dzongkha (13)} + \text{Maths (5)} = 32$

The required probability = $P(A) = \frac{n(A)}{n(S)} = \frac{32}{100} = 0.32$

Ans: D 0.32

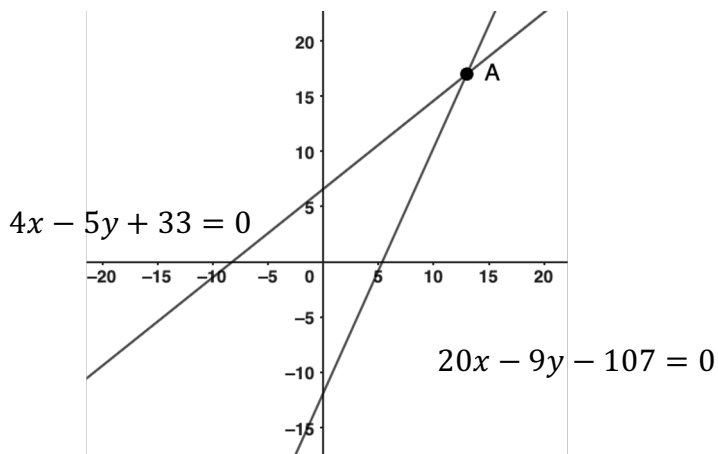
vi. Integrate: $\int \frac{3x-2}{(3x^2+x-2)} dx$									
<p>A $\frac{x^2}{2} + x + c$</p> <p>B $\frac{x^2}{2} - x + c$</p> <p><input checked="" type="radio"/> C $\log x + 1 + c$</p> <p>D $\frac{1}{\log} x + 1 + c$</p>									
<p>$3x^2 + x - 2 = (3x - 2)(x + 1)$</p> <p>$\int \frac{3x-2}{(3x-2)(x+1)} dx = \int \frac{1}{x+1} dx = \log x+1 + C$</p> <p>Ans: C $\log x + 1 + c$</p>									
vii. Mr. Karma Rangdrol wishes to create a fund to provide a cash prize of Nu 1953 for the first rank holder in RCSC preliminary examinations every year. If the fund is invested at 4% p.a. compound interest, the amount invested is									
<p>A Nu 3126.81.</p> <p>B Nu 18,329.04.</p> <p>C Nu 29,345.39.</p> <p><input checked="" type="radio"/> D Nu 48,825.00.</p>									
<p>Given that:</p> <p>$a = 1953, i = 4\% = 0.04$</p> <p>Then,</p> <p>$P = \frac{a}{i} = \frac{1953}{0.04} = 48,825.00$</p> <p>Ans: D Nu 48,825.00</p>									
viii. The correct pairs from the following table are									
<table border="1"> <thead> <tr> <th>Conditions</th> <th>Turning points</th> </tr> </thead> <tbody> <tr> <td>I. $\frac{d^2y}{dx^2} > 0$</td> <td>a. Point of inflexion</td> </tr> <tr> <td>II. $\frac{d^2y}{dx^2} < 0$</td> <td>b. Maximum point</td> </tr> <tr> <td>III. $\frac{d^3y}{dx^3} \neq 0$</td> <td>c. Minimum point</td> </tr> </tbody> </table>	Conditions	Turning points	I. $\frac{d^2y}{dx^2} > 0$	a. Point of inflexion	II. $\frac{d^2y}{dx^2} < 0$	b. Maximum point	III. $\frac{d^3y}{dx^3} \neq 0$	c. Minimum point	
Conditions	Turning points								
I. $\frac{d^2y}{dx^2} > 0$	a. Point of inflexion								
II. $\frac{d^2y}{dx^2} < 0$	b. Maximum point								
III. $\frac{d^3y}{dx^3} \neq 0$	c. Minimum point								

- (A) I → c, II → b, III → a.
 B I → b, II → c, III → a.
 C I → a, II → c, III → b.
 D I → c, II → a, III → b.

Conditions	Turning points
I. $\frac{d^2y}{dx^2} > 0$	a. Point of inflexion
II. $\frac{d^2y}{dx^2} < 0$	b. Maximum point
III. $\frac{d^3y}{dx^3} \neq 0$	c. Minimum point

Ans: A I → c, II → b, III → a

ix. Find the intersection point of the regression lines from the graph.



- A $x = 13.25, y = 17.25$
 B $x = 13.5, y = 17.5$
 C $x = 14, y = 18$
 (D) $x = 13, y = 17$

$$4x - 5y = -33$$

$$20x - 9y = 107$$

$$20x - 25y = -165$$

$$20x - 9y = 107$$

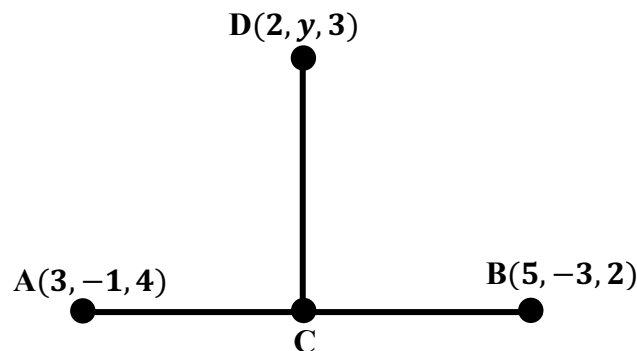
$$-16y = -272 \Rightarrow y = 17$$

$$4x - 5(17) = -33 \Rightarrow 4x = 52 \Rightarrow x = 13$$

Therefore, the intersection point of the regression lines is $x = 13$ and $y = 17$

Ans: D $x = 13, y = 17$

x. The point C divides the line segment AB in the ratio 2:2. Find the missing coordinate of point D if the two lines AB is perpendicular to CD.



A -6

B -4

C 4

D 6

The point C is the mid-point of AB.

The coordinates of the point C(4, -2, 3).

Here, the d.r.s of AB is $a_1 = 2, b_1 = -2, c_1 = -2$

And, the d.r.s of CD is $a_2 = -2, b_2 = y + 2, c_2 = 0$

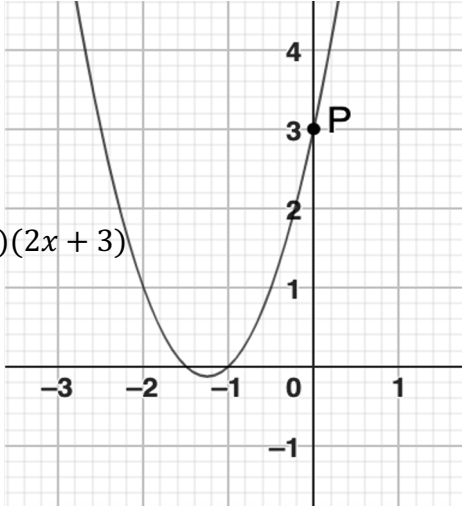
Since AB is perpendicular to CD, $\sum a_1 a_2 = 0$.

$$2 \times (-2) + (-2)(y + 2) + 0 = 0$$

$$-4 - 2y - 4 = 0$$

$$y = \frac{8}{-2} = -4$$

Ans: B -4

<p>xi. In EURO 2024 football tournament, each team played with every other team exactly once. If the total number of matches played is 36, how many teams participated in the tournament?</p>	
<p>A 72 teams B 36 teams C 24 teams Ⓓ 9 teams</p>	
<p>${}^n C_2 = 36 \Rightarrow \frac{n!}{2!(n-2)!} = 36 \Rightarrow \frac{n(n-1)(n-2)!}{(n-2)!} = 72$ $n(n-1) = 72 \Rightarrow n(n-1) = 9.8 \Rightarrow 9.8 = 9.8 \Rightarrow n = 9$ Therefore, the number of teams in the matches were 9. Ans: D 9 teams</p>	
<p>xii. A mathematics teacher used GeoGebra app to draw a graph of a function. A random variable point P was plotted on a graph as shown below. The teacher wants the students to calculate the slope at point P. What is the slope at the given point?</p> <div style="text-align: center;">  <p>$f(x) = (x + 1)(2x + 3)$</p> </div>	
<p>A 17 B 8 Ⓒ 5 D 3</p>	

$$y = (x+1)(2x+3) = 2x^2 + 5x + 3$$

$$\frac{dy}{dx} = 4x + 5 \Rightarrow \frac{dy}{dx} \Big|_{(0,3)} = 4(0) + 5 = 0 + 5 = 5$$

\therefore the slope (m) = 5

Ans: C 5

xiii. An amateur entrepreneur visited his friend to seek financial suggestions on a setup of new business. He was provided with the following profit functions. What would be the best choice to generate maximum profit when $x = 10$?

- (A) $-1300 + 400x - 20x^2$
- B $-x^2 + 20x + 350$
- C $133 - 45x + 5x^2$
- D $\frac{15}{x^2} + 30x + 250$

A $P(x) = -1300 + 400x - 20x^2$

$$\frac{dP}{dx} = 400 - 4x$$

$$\frac{d^2P}{dx^2} = -4 < 0 \text{ (maximum)}$$

When $x = 10, P(10) = -1300 + 4000 - 2000 = 700$

B $P(x) = -x^2 + 20x + 350$

$$\frac{dP}{dx} = -2x + 20$$

$$\frac{d^2P}{dx^2} = -2 < 0 \text{ (maximum)}$$

When $x = 10, P(10) = 100 + 200 + 350 = 650$

Option C and D are inadmissible because the functions give minimum points.

Ans: A $-1300 + 400x - 20x^2$

xiv. The table below shows the rank differences of Economics and History. Calculate the correlation coefficient between the two subjects.

Sl.No	1	2	3	4	5	6	7	8	9	10
D	-1	-2	2	-1	2	-3	0	-2	x	2

- A 1.00
- B 0.75
- C -0.75
- D -1.00

To find the value of x , we need to ensure that the sum of all rank differences is zero.

Sl.No	1	2	3	4	5	6	7	8	9	10
D	-1	-2	2	-1	2	-3	0	-2	3	2
D^2	1	4	4	1	4	9	0	4	9	4

$$\sum D = 0 \Rightarrow -9 + 6 + x = 0$$

$$-3 + x = 0 \Rightarrow x = +3$$

We know,

$$r = 1 - \frac{6 \sum D^2}{n(n^2-1)}$$

$$= 1 - \frac{6 \times 40}{10(10^2-1)}$$

$$= 1 - \frac{240}{10(99)}$$

$$r = \frac{750}{990} = 0.75$$

Ans: B 0.75

xv. What is $\det(A^{-1})$, if A is an invertible matrix?

- A $\det(A)$
- B $\frac{1}{\det(A)}$
- C I_n
- D 0

By using the theorem $AA^{-1} = I$, multiply both sides by a determinant.

Now, we get

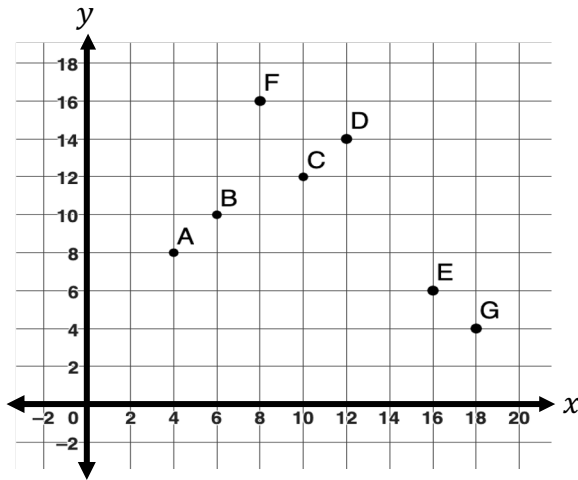
$\det(AA^{-1}) = \det(I)$ $\det A \cdot \det A^{-1} = \det(I)$ $\det(A^{-1}) = \frac{1}{\det(A)} \quad [\because \det(I) = 1]$ Ans: B $\frac{1}{\det(A)}$	
---	--

SECTION B (70 MARKS)

ANSWER ANY TEN QUESTIONS

Question 2

a) If $\begin{bmatrix} x & y \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} y & -x \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 11 \\ 5 & 0 \end{bmatrix}$, then find the missing elements.	[3]
<p>Ans: $\begin{bmatrix} x & y \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} y & -x \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 11 \\ 5 & 0 \end{bmatrix}$</p> $\begin{bmatrix} x+y & y-x \\ 3+2 & -1+1 \end{bmatrix} = \begin{bmatrix} 5 & 11 \\ 5 & 0 \end{bmatrix}$ $\begin{bmatrix} x+y & y-x \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} 5 & 11 \\ 5 & 0 \end{bmatrix}$ <p>From the matrix equation, we get</p> $x + y = 5 \dots (1)$ $-x + y = 11 \dots (2)$ <p>From (1), we get $y = 5 - x \dots (3)$</p> <p>Put (3) into (2), we get</p> $-x + 5 - x = 11 \Rightarrow -2x = 6 \Rightarrow x = -3$ <p>Put $x = -3$ into (3), we get</p> $y = 5 + 3 = 8$ <p>Here, $x = -3$ and $y = 8$.</p>	<p>0.5</p> <p>0.5</p> <p>0.5</p> <p>0.5</p> <p>0.5</p> <p>0.5</p>
b) The following graph depicts the test scores of 7 students in Dzongkha (x) and English (y).	[4]



A student scored 5 marks in Dzongkha but was absent from the English test.
Estimate his probable score for the later test.

Ans:

x	y	x^2	y^2	xy
4	8	16	64	32
6	10	36	100	60
8	16	64	256	128
10	12	100	144	120
12	14	144	196	168
16	6	256	36	96
18	4	324	16	72
74	70	940	812	676

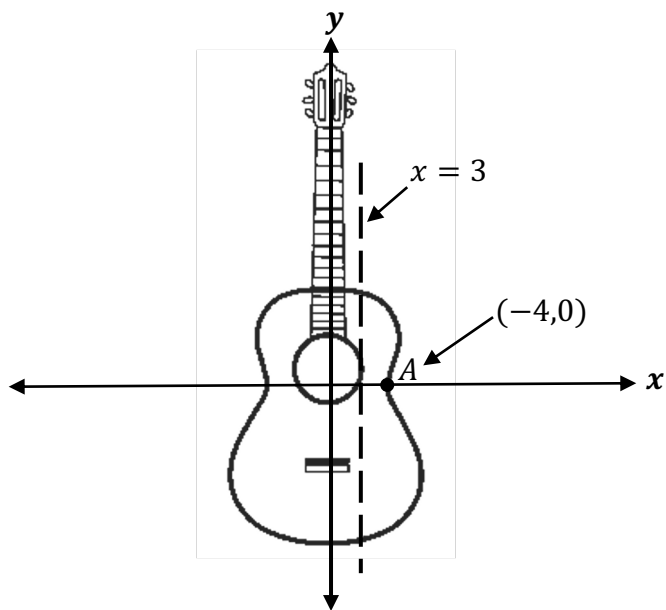
We get,

$$\bar{x} = \frac{\sum x}{n} = \frac{74}{7} = 10.57, \bar{y} = \frac{\sum y}{n} = \frac{70}{7} = 10$$

We know,

$$b_{yx} = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}} = \frac{676 - \frac{74 \times 70}{7}}{940 - \frac{(74)^2}{7}} = \frac{676 - 740}{940 - 782.29} = \frac{-64}{157.71}$$

<p>$= -0.41$</p> <p>The regression line of y on x is</p> $y - \bar{y} = b_{yx}(x - \bar{x})$ $y - 10 = -0.41(x - 10.57)$ $y = -0.41x + 14.33$ <p>When $x = 5$, then $y = -0.41(5) + 14.33 = 12.28$</p> <p>Therefore, his probable score in English is about 12.</p>	<p>0.5</p> <p>0.5</p> <p>0.5</p>	
<p>Question 3</p>		
<p>a) The marginal cost function of manufacturing x units of a product is $5 + 16x - 3x^2$. The total cost of producing 5 units is Nu 500. Find the total cost function.</p>	<p>[3]</p>	
<p>Ans: Given that,</p> $MC = 5 + 16x - 3x^2$ <p>Then, the total cost is</p> $\int MC dx = C(x) + c$ <p>So,</p> $\int (5 + 16x - 3x^2) dx = 5x + \frac{16x^2}{2} - \frac{3x^3}{3} + c$ $C(x) = 5x + 8x^2 - x^3 + c$ <p>Also, $C(x) = 500$, when $x = 5$, then</p> $500 = 5 \times 5 + 8(5)^2 - (5)^3 + c$ $500 = 25 + 8 \times 25 - 125 + c$ $500 = 200 - 100 + c$ $c = 400$ <p>Therefore, the total cost function is $5x + 8x^2 - x^3 + 400$</p>	<p>0.5</p> <p>0.5</p> <p>0.5</p> <p>0.5</p> <p>0.5</p> <p>0.5</p>	
<p>b) Determine the type of the conic section whose directrix and vertex A are given.</p>	<p>[4]</p>	



Find the following:

- | | |
|-------------------------------------|------------|
| i. Coordinates of foci | [1] |
| ii. Eccentricity | [1] |
| iii. Equation of the conic section. | [2] |

Ans:

i. Coordinates of foci: $(\pm ae, 0) = \left(\pm \frac{16}{3}, 0\right)$ } **1**

ii. Eccentricity:

$$a = 4, \frac{a}{e} = 3, \frac{4}{e} = 3$$

$$e = \frac{4}{3}$$

} **0.5**
} **0.5**

iii. Equation of a conic section:

$$\therefore ae = \frac{16}{3} \Rightarrow a = 4$$

$$\therefore b^2 = a^2(e^2 - 1) = \frac{16(16-9)}{9} = \frac{112}{9}$$

$$\frac{x^2}{16} - \frac{y^2}{112/9} = 1$$

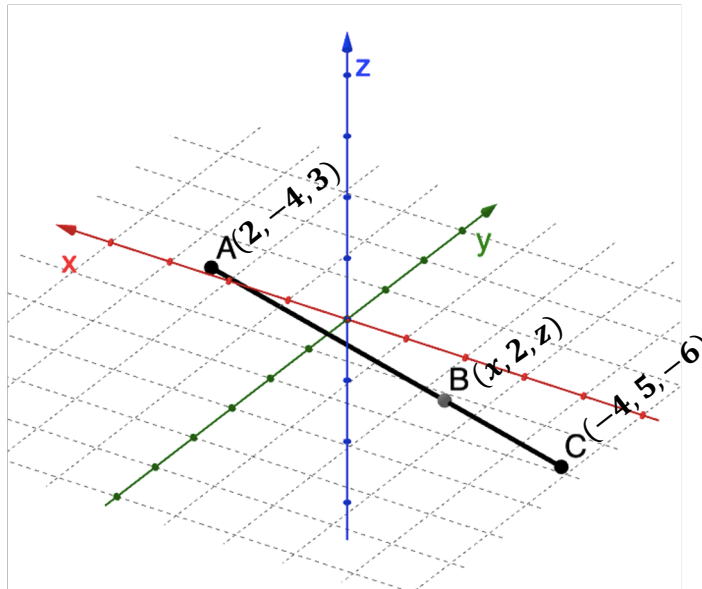
$$112x^2 - 144y^2 = 1792$$

} **0.5**
} **0.5**

Question 4

- a) Ms. Rigzin Om plots the three points on the cartesian coordinate system as shown below. However, she could not find two coordinates of the point B which is two-thirds of the way from AC. Find the missing coordinates.

[3]



Ans: Using the section formula, we get

$$x = \frac{mx_1 + nx_2}{m+n}$$

$$\Rightarrow \frac{2(-4) + 1(2)}{2+1} = \frac{-8+2}{3} = \frac{-6}{3} = -2$$

$$z = \frac{mz_1 + nz_2}{m+n}$$

$$\Rightarrow \frac{2(-6) + 1(3)}{2+1} = \frac{-12+3}{3} = \frac{-9}{3} = -3$$



0.5

1

0.5

1

- b) Mr. Karma set up a dairy farm in his village. He borrowed Nu 30,000 from Bhutan Development Bank Limited on a condition to repay it with compound interest at 5% p.a. at the annual installments of Nu 4,017. In how many years will his debt be liquidated?

[4]

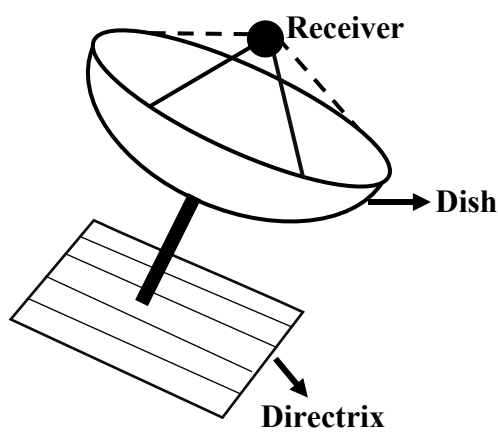
Ans: Given that:

$P = Nu\ 30,000, \quad a = Nu\ 4017, \quad i = 0.05 \quad \text{and} \quad n = ?$ $Now, P = \frac{a}{i} [1 - (1+i)^{-n}]$ $\Rightarrow 30000 = \frac{4017}{0.05} [1 - (1+0.05)^{-n}]$ $30000 = \frac{4017}{0.05} [1 - (1+0.05)^{-n}]$ $\Rightarrow \frac{30000 \times 0.05}{4017} = [1 - (1.05)^{-n}]$ $0.3734 = 1 - (1.05)^{-n} \Rightarrow -0.6266 = -(1.05)^{-n}$ $\log(0.6266) = -\log(1.05) \Rightarrow -0.2030 = -0.0212n$ $n = \frac{0.2030}{0.0212} = 9.575 \approx 10 \text{ years}$	} 0.5 } 0.5 } 0.5 } 0.5 } 1 } 1	
Question 5		
a) Ms. Pema Dema was playing a game to form different words using the letters of her last name D, E, M, A. She could form 4! different words. Then, she was excited to try with a different word 'MATHEMATICS'. Find the number of combinations of four letters taken from the word MATHEMATICS.		[3]
Ans: Here, M(2 times), A(2 times), T(2 times), H,E,I,C,S	}	0.5
The possible selections are:		
i. All four letters are different ($n = M, A, T, H, E, I, C, S, r = 4$) The no. of selection is ${}^8C_4 = 70$ ways	}	0.5
ii. Two alike, two alike ($n = MM, AA, TT, r = 2$) The no. of ways is ${}^3C_2 = 3$ ways	}	0.5
iii. Two alike, 2 different Two likes ($n = MM, AA, TT, r = 2$) 2 different ($n = A, T, H, E, I, C, S, r = 2$) The no. of ways is ${}^3C_1 \times {}^7C_2 = 3 \times 21 = 63$ ways	}	0.5
Therefore, the total combinations are $70 + 3 + 63 = 136$ ways	}	0.5
b) Integrate: $\int \frac{10x^2+9x-7}{(x+2)(x^2-1)} dx$		[4]

<p>Ans:</p> $\int \frac{10x^2 + 9x - 7}{(x+2)(x^2 - 1)} dx$ $\Rightarrow \frac{10x^2 + 9x - 7}{(x+2)(x^2 - 1)} = \frac{10x^2 + 9x - 7}{(x+2)(x-1)(x+1)}$ $\Rightarrow \frac{10x^2 + 9x - 7}{(x+2)(x-1)(x+1)} = \frac{A}{x+2} + \frac{B}{x-1} + \frac{C}{x+1}$ $\Rightarrow \frac{A(x-1)(x+1) + B(x+2)(x+1) + C(x+2)(x-1)}{(x+2)(x-1)(x+1)}$ <p>Put $x = 1$, $10(1)^2 + 9(1) - 7 = A(1-1)(1+1) + B(1+2)(1+1) + C(1+2)(1-1)$ $\Rightarrow 12 = 6B \Rightarrow B = 2$</p> <p>Put $x = -1$, $10(-1)^2 + 9(-1) - 7 = A(-1-1)(-1+1) + B(-1+2)(-1+1) + C(-1+2)(-1-1)$ $\Rightarrow -6 = -2C \Rightarrow C = 3$</p> <p>Put $x = -2$, $10(-2)^2 + 9(-2) - 7 = A(-2-1)(-2+1) + B(-2+2)(-2+1) + C(-2+2)(-2-1)$ $\Rightarrow 15 = 3A \Rightarrow A = 5$</p> <p>Now, $\int \left(\frac{5}{x+2} + \frac{2}{x-1} + \frac{3}{x+1} \right) dx \Rightarrow 5 \log x+2 + 2 \log x-1 + 3 \log x+1 + C$</p>	<p>} 0.5</p> <p>} 0.5</p> <p>} 0.5</p> <p>} 0.5</p> <p>} 0.5</p> <p>} 1</p>	<p>0.5</p> <p>0.5</p> <p>0.5</p> <p>0.5</p> <p>0.5</p> <p>1</p>
--	---	---

Question 6

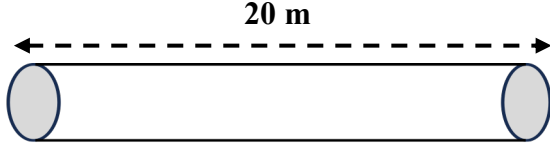
a) A satellite dish has its receiver located at the focus. **[3]**



i. How far is the receiver from the vertex if the dish is 10ft across and 3ft deep at the center? **[1.5]**

ii. Find the equation of the parabola. **[1.5]**

Ans:																																												
Let the equation of ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	}	0.5																																										
Now, $x^2 + y^2 = 5^2$ is the required equation of the ellipse.	}	0.5																																										
Differentiate on both sides, we get	}																																											
$\frac{d}{dx}x^2 + \frac{d}{dx}y^2 = \frac{d}{dx}5^2$	}	0.5																																										
$2x + 2y\frac{dy}{dx} = 0$	}	0.5																																										
$2y\frac{dy}{dx} = -2x$	}	0.5																																										
$\frac{dy}{dx} = -\frac{2x}{2y} = -\frac{x}{y}$	}	0.5																																										
Now, the gradient of a ellipse is																																												
$\frac{dy}{dx} / (3,4) = -\frac{x}{y} = -\frac{3}{4}$	}	0.5																																										
Question 7																																												
a) Imagine you are managing quality control for a factory that produces light bulbs. The checklist of tested bulbs is recorded in a table with tick mark as shown below. You randomly select one bulb to test.		[3]																																										
<table border="1"> <tr> <td>Sl. No</td> <td>1</td> <td>2</td> <td>3</td> <td>4</td> <td>5</td> <td>6</td> <td>7</td> <td>8</td> <td>9</td> <td>10</td> <td>11</td> <td>12</td> <td>13</td> </tr> <tr> <td>Defective</td> <td></td> <td></td> <td>✓</td> <td></td> <td></td> <td>✓</td> <td>✓</td> <td>✓</td> <td></td> <td></td> <td></td> <td>✓</td> <td>✓</td> </tr> <tr> <td>Not defective</td> <td>✓</td> <td>✓</td> <td></td> <td>✓</td> <td>✓</td> <td></td> <td></td> <td></td> <td>✓</td> <td>✓</td> <td>✓</td> <td></td> <td></td> </tr> </table>		Sl. No	1	2	3	4	5	6	7	8	9	10	11	12	13	Defective			✓			✓	✓	✓				✓	✓	Not defective	✓	✓		✓	✓				✓	✓	✓			
Sl. No	1	2	3	4	5	6	7	8	9	10	11	12	13																															
Defective			✓			✓	✓	✓				✓	✓																															
Not defective	✓	✓		✓	✓				✓	✓	✓																																	
i. What is the probability that the bulb you selected is not defective?		[1.5]																																										
ii. The bulb selected is defective and it is not replaced, and a second bulb is selected at random from the rest, what is the probability that the second bulb is defective?		[1.5]																																										
Ans:																																												

<p>i. $n(S) = 13$</p> <p>$E = \text{Not defective light bulb}; n(E) = 7$</p> $P(E) = \frac{n(E)}{n(S)} = \frac{7}{13}$ <p>ii. $n(S) = 13 - 1 = 12$</p> <p>$F = \text{Defective light bulb}; n(F) = 6 - 1 = 5$</p> $P(F) = \frac{n(F)}{n(S)} = \frac{5}{12}$	<div style="text-align: right;"> $\left. \begin{array}{l} \text{---} \\ \text{---} \\ \text{---} \end{array} \right\}$ $\left. \begin{array}{l} \text{---} \\ \text{---} \end{array} \right\}$ $\left. \begin{array}{l} \text{---} \\ \text{---} \\ \text{---} \end{array} \right\}$ $\left. \begin{array}{l} \text{---} \\ \text{---} \end{array} \right\}$ </div> <p style="text-align: right;">0.5</p> <p style="text-align: right;">1</p> <p style="text-align: right;">0.5</p> <p style="text-align: right;">1</p>
<p>b) A house proprietor brought a 12mm thick iron rod. He wants to cut the rod into two parts so that the product of the square of the one and the cube of the other may be the greatest possible. Find the length of two parts of the iron rod.</p> <div style="text-align: center;">  </div>	<p style="text-align: right;">[4]</p>
<p>Ans: Let the length of two parts of an iron rod be x and y.</p> <p>Then, $x + y = 20 \dots (1)$</p> <p>Let P be the product of the square of the one and the cube of the other may be the greatest possible.</p> <p>So, $P = x^3 y^2 \dots (2)$</p> <p>From (1), we get</p> $x + y = 20 \Rightarrow y = 20 - x \dots (3)$ <p>Put (3) into (2), we get</p> $P = x^3 (20 - x)^2$ $= x^3 (400 - 40x + x^2)$ $= 400x^3 - 40x^4 + x^5$ <p>Then, $\frac{dP}{dx} = 1200x^2 - 160x^3 + 5x^4$</p>	<div style="text-align: right;"> $\left. \begin{array}{l} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right\}$ $\left. \begin{array}{l} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right\}$ $\left. \begin{array}{l} \text{---} \\ \text{---} \\ \text{---} \end{array} \right\}$ </div> <p style="text-align: right;">0.5</p> <p style="text-align: right;">0.5</p> <p style="text-align: right;">0.5</p>

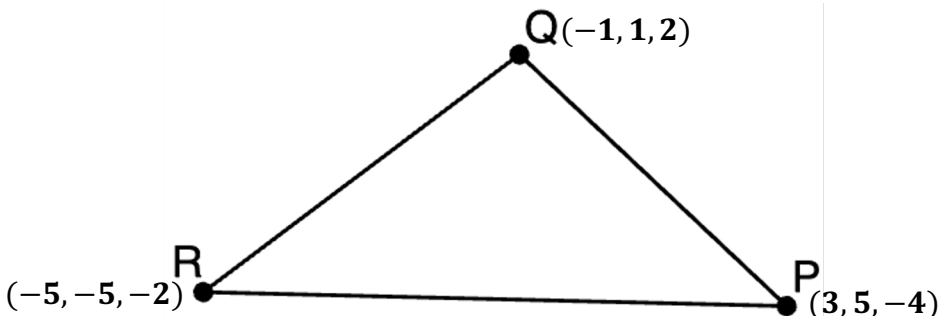
$60 \times \left(1 + \frac{2i}{3}\right) = 1860 \times \frac{2i}{3}$	}	0.5	
$60 + \frac{120i}{3} = 1240i$			
$60 + 40i = 1240i$	}	0.5	
$60 = 1240i - 40i$			
$1200i = 60$	}	0.5	
$i = 0.05 \times 100 = 5\%$			

<p>b) Verify that $\frac{d^2y}{dx^2} = \frac{1}{8}$, when $t = 1$.</p>	[4]

Ans:	
-------------	--

$\frac{dx}{dt} = 3t^2 - 10t + 3$	}	0.5
$\frac{dy}{dt} = 2t - 2$		
$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2t - 2}{3t^2 - 10t + 3}$		
$\frac{d^2y}{dx^2} = \frac{d}{dt} \left(\frac{2t - 2}{3t^2 - 10t + 3} \right) \cdot \frac{dt}{dx}$		
$\Rightarrow \frac{(3t^2 - 10t + 3)2 - (2t - 2)(6t - 10)}{(3t^2 - 10t + 3)^2} \cdot \frac{1}{3t^2 - 10t + 3}$		
$\Rightarrow \frac{(-6t^2 + 12t - 14)}{(3t^2 - 10t + 3)^2} \cdot \frac{1}{3t^2 - 10t + 3}$		
$= \frac{-6t^2 + 12t - 14}{(3t^2 - 10t + 3)^3}$	}	0.5
When $t = 1$, $\frac{d^2y}{dx^2} \Big _{t=1} = \frac{-6(1) + 12(1) - 14}{[3(1)^2 - 10(1) + 3]^3}$		
$\Rightarrow \frac{-6 + 12 - 14}{(3 - 10 + 3)^3} = \frac{-8}{-64} = \frac{1}{8}$		

Question 9

a) Show that the given polygon is an isosceles triangle.	[3]
	

<p>Ans: Using the distance formula,</p> $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$ <p>we get</p> $PQ^2 = (-1 - 3)^2 + (1 - 5)^2 + (2 + 4)^2$ $= 16 + 16 + 36$ $= 68$ $QR^2 = (-5 + 1)^2 + (-5 - 1)^2 + (2 + 2)^2$ $= 16 + 36 + 16$ $= 68$ <p>Here, $PQ = QR$, therefore PQR is an isosceles triangle.</p>	<p style="text-align: right;">}</p> <p>0.5</p> <p style="text-align: right;">}</p> <p>0.5</p> <p style="text-align: right;">}</p> <p>0.5</p> <p style="text-align: right;">}</p> <p>0.5</p> <p style="text-align: right;">}</p> <p>0.5</p>
<p>b) The cost function of producing x units of a product is given by $C(x) = \sqrt{ax + b}$. Show how the nature of the average cost curve continuously changes with increasing output when both a and b are positive and when they are negative?</p>	<p style="text-align: right;">[4]</p>
<p>Ans:</p> $AC = \frac{C(x)}{x} = \frac{\sqrt{ax + b}}{x} = \frac{(ax + b)^{\frac{1}{2}}}{x}$ <p>When both a and b are positive, if $a = 2$, $b = 4$ and $x = 1, 10, 100$, the average cost are:</p> $AC(1) = \frac{\sqrt{2(1) + 4}}{1} = \sqrt{6} = 2.4$ $AC(10) = \frac{\sqrt{2(10) + 4}}{10} = \frac{\sqrt{24}}{10} = 0.49$ $AC(100) = \frac{\sqrt{2(100) + 4}}{100} = \frac{\sqrt{204}}{100} = 0.14$ <p>When both a and b are negative, if $a = -2$, $b = -4$ and $x = 1, 10, 100$, the average cost are:</p>	<p style="text-align: right;">}</p> <p>0.5</p> <p style="text-align: right;">}</p> <p>0.5</p> <p style="text-align: right;">}</p> <p>0.5</p> <p style="text-align: right;">}</p> <p>0.5</p>

$$AC(1) = \frac{\sqrt{-2(1)-4}}{1} = \sqrt{-6} = \sqrt{6}i = 2.4i$$

$$AC(10) = \frac{\sqrt{-2(10)-4}}{10} = \frac{\sqrt{-24}}{10} = \frac{\sqrt{24}i}{10} = 0.49i$$

$$AC(100) = \frac{\sqrt{-2(100)-4}}{100} = \frac{\sqrt{-204}}{100} = \frac{\sqrt{204}i}{100} = 0.14i$$

Therefore, the average cost curve falls continuously when the signs of both constants a and b changes and increases the outputs.

Question 10

a) A school administrator wants to determine if there is a significant relationship between students' study time (in hours) and examination scores. **[3]**

Study time (in hours)	5	3	2	6	1	4
Examination scores	72	50	45	80	25	60

What does it suggest about the relationship between students' study time and examination scores?

Ans:

Study time in hours (x)	Examination Scores (y)	x²	y²	xy
5	72	25	5184	360
3	50	9	2500	150
2	45	4	2025	90
6	80	36	6400	480
1	25	1	625	25
4	60	16	3600	240
$\sum x = 21$	$\sum y = 332$	$\sum x^2 = 91$	$\sum y^2 = 20,334$	$\sum xy = 1345$

}
}
}

0.5
0.5
0.5

$r = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sqrt{\sum x^2 - \frac{(\sum x)^2}{n}} \sqrt{\sum y^2 - \frac{(\sum y)^2}{n}}}$ $= \frac{1345 - \frac{21 \times 332}{6}}{\sqrt{91 - \frac{(21)^2}{6}} \sqrt{20,334 - \frac{(332)^2}{6}}}$ $= \frac{1345 - 1162}{\sqrt{91 - 74} \cdot \sqrt{20,334 - 18,371}}$ $= \frac{183}{\sqrt{17} \sqrt{1963}}$ $\Rightarrow \frac{183}{4.1 \times 44.3} = \frac{183}{181.63} = 1.007 \approx 1$ <p>From the value $r = 1$, it can be concluded that the more hours spent studying tends to the higher test scores.</p>	<div style="display: flex; flex-direction: column; align-items: center;"> <div style="margin-bottom: 10px;">}</div> <div style="margin-bottom: 10px;">}</div> <div style="margin-bottom: 10px;">}</div> <div style="margin-bottom: 10px;">}</div> <div style="margin-bottom: 10px;">}</div> </div>
<p>b) Integrate the following functions:</p>	[4]
<p>i. $\frac{7x^3 - 6x}{\sqrt{x}}$</p>	[2]
<p>ii. $\frac{6x+5}{\sqrt{3x^2+5x+2}}$</p>	[2]
<p>Ans:</p> <p>i. $\int \frac{7x^3 - 6x}{\sqrt{x}} dx = \int \frac{7x^3 - 6x}{x^{\frac{1}{2}}} dx$</p> $= \int 7x^{3-\frac{1}{2}} - 6x^{1-\frac{1}{2}} dx$ $= \int 7x^{\frac{5}{2}} - 6x^{\frac{1}{2}} dx$ $= \frac{7x^{\frac{5}{2}+1}}{\frac{5}{2}+1} - \frac{6x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + c$ $= \frac{7x^{\frac{7}{2}}}{\frac{7}{2}} - \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} + c$ $= 2x^{\frac{7}{2}} - 4x^{\frac{3}{2}} + c$	<div style="display: flex; flex-direction: column; align-items: center;"> <div style="margin-bottom: 10px;">}</div> <div style="margin-bottom: 10px;">}</div> <div style="margin-bottom: 10px;">}</div> <div style="margin-bottom: 10px;">}</div> </div>

ii. $\int \frac{6x+5}{\sqrt{3x^2+5x+2}} dx$ $Let\ t = 3x^2 + 5x + 2 \Rightarrow \frac{dt}{dx} = 6x + 5$ $dx = \frac{1}{6x+5} dt$ $\therefore \int \frac{6x+5}{\sqrt{3x^2+5x+2}} dx = \int \frac{6x+5}{\sqrt{t}} \times \frac{1}{6x+5} dt$ $= \int \frac{1}{\sqrt{t}} dt$ $= \int t^{-\frac{1}{2}} dt$ $= 2t^{\frac{1}{2}} + c$ $= 2\sqrt{3x^2 + 5x + 2} + c$	$\left. \begin{array}{l} \\ \\ \\ \\ \\ \end{array} \right\}$ $\left. \begin{array}{l} \\ \\ \\ \end{array} \right\}$ $\left. \begin{array}{l} \\ \\ \end{array} \right\}$ $\left. \begin{array}{l} \\ \end{array} \right\}$	0.5 0.5 0.5 0.5
---	---	--

Question 11

a) Two dealers are selling the same model of a refrigerator. b)	[3]						
<table border="1" style="margin: auto;"> <thead> <tr> <th></th> <th>Wangchuk's Shop</th> <th>Sonam's Shop</th> </tr> </thead> <tbody> <tr> <th>Discount Offer</th> <td>Gives successive discounts of 25% and 5%</td> <td>Gives successive discounts of 16% and 14%</td> </tr> </tbody> </table> <p>As a customer, which one would you prefer?</p>		Wangchuk's Shop	Sonam's Shop	Discount Offer	Gives successive discounts of 25% and 5%	Gives successive discounts of 16% and 14%	
	Wangchuk's Shop	Sonam's Shop					
Discount Offer	Gives successive discounts of 25% and 5%	Gives successive discounts of 16% and 14%					

Ans: Wangchuk's Shop Let MP be Nu 100 Less 25% discount = $\frac{25}{100} \times 100 = 25$ Sale price = $100 - 25 = 75$ Less 5% discount = $\frac{5}{100} \times 75 = 3.75$ Sale price = $75 - 3.75 = 71.25$ Effective discount = $100 - 71.25 = 28.75\%$ Sonam's Shop Let MP be Nu 100 Less 16% discount = $\frac{16}{100} \times 100 = 16$ Sale price = $100 - 16 = 84$ Less 12% discount = $\frac{12}{100} \times 84 = 10.08$	$\left. \begin{array}{l} \\ \\ \\ \\ \\ \end{array} \right\}$ $\left. \begin{array}{l} \\ \\ \end{array} \right\}$	0.5 0.5 0.5 0.5
---	---	--

<p>Sale price = $84 - 10.08 = 73.92$ Effective discount = $100 - 73.92 = 26.08\%$ Wangchuk's shop offers a higher effective discount of 28.75% compared to Sonam's 26.08%.</p>	} }	0.5
<p>Therefore, Wangchuk's shop is more advantageous to the customer.</p>	} }	0.5
c) The school management team (SMT) is comprising of the Principal, the Vice Principal, Academic Head, Student Support Service, and Staff Secretary. In the meeting, they sit around a round table.		[4]
i. In how many ways can they be seated?		[1]
ii. In how many ways can they be seated if the Principal and Vice Principal choose to sit next to each other?		[1]
iii. If there are 4 gentlemen and 2 ladies in SMT, in how many ways 4 gentlemen and 2 ladies can be seated round a table so that every gentleman may have a lady by his side?		[2]
Ans:		
i. Here, $n = 5$ The number of ways = $(5 - 1)!$ $= 4! = 24$	} } }	0.5 0.5
ii. Since, the 'Principal' and the 'Vice Principal' sit together and it is considered as one. Then, $n = 4$ The 'Principal' and 'Vice Principal' can sit together in 2 ways. Therefore, the total number of ways = $(4 - 1)! 2!$ $= 2(3)!$ $= 12$	} } }	0.5 0.5
iii. Arrange the seats for 4 gentlemen and 2 ladies as shown. The arrangement can be done in $(4 - 1)!$ (gentlemen) and $2!$ (ladies) ways. The number of ways = $3! \times 2! = 12$ ways If the arrangements are made in the opposite direction, then the number of arrangements = $3! \times 2! = 12$ ways Therefore, total arrangements = $12 + 12$ $= 24$ ways	} } } } }	0.5 0.5 0.5 0.5

Question 12	
a) Create a system of two linear equations that have no solution without repeating the elements.	[3]
<p>Let two linear equations be $2p + 5q = 7$ and $6p + 15q = 13$</p> <p>From the equations, we have $A = \begin{bmatrix} 2 & 5 \\ 6 & 15 \end{bmatrix}$, $X = \begin{bmatrix} p \\ q \end{bmatrix}$ and $B = \begin{bmatrix} 7 \\ 13 \end{bmatrix}$</p> <p>$A = \begin{vmatrix} 2 & 5 \\ 6 & 15 \end{vmatrix} = 30 - 30 = 0$</p> <p>$\therefore A = 0$, the inverse does not exist.</p> <p>$adjA = \begin{bmatrix} 15 & -5 \\ -6 & 2 \end{bmatrix}$</p> <p>Now, $(adjA)B = \begin{bmatrix} 15 & -5 \\ -6 & 2 \end{bmatrix} \begin{bmatrix} 7 \\ 13 \end{bmatrix}$</p> <p>$\Rightarrow \begin{bmatrix} (15 \times 7) + (-5 \times 13) \\ (-6 \times 7) + (2 \times 13) \end{bmatrix} = \begin{bmatrix} 105 - 65 \\ -42 + 26 \end{bmatrix} = \begin{bmatrix} 40 \\ -16 \end{bmatrix}$</p> <p>$\therefore (adjA)B \neq \begin{bmatrix} 40 \\ -16 \end{bmatrix}$, therefore, the system of equations has no solution.</p>	<p>0.5</p> <p>0.5</p> <p>0.5</p> <p>0.5</p> <p>0.5</p> <p>0.5</p>
<p>b) Bank of Bhutan officially announced the new year offers on annuity scheme as shown in the tree diagram. Study the offers and answer the following questions.</p> <div style="text-align: center; margin-top: 20px;"> <div style="border: 1px solid black; padding: 5px; display: inline-block; width: 40%;"> <p>Consider an ordinary annuity where regular payments of Nu 500 are made at the end of each year for 10 years at an interest rate of 6% per annum.</p> </div> <div style="display: inline-block; vertical-align: middle; margin-left: 10px;"> <p style="font-size: 2em;">}</p> </div> <div style="border: 1px solid black; padding: 5px; display: inline-block; width: 30%; margin-left: 10px;"> <p>Future value of the annuity if the annual interest rate is increased to 8%</p> </div> <div style="border: 1px solid black; padding: 5px; display: inline-block; width: 30%; margin-left: 10px;"> <p>Future value of the annuity if the number of duration increased to 15 years.</p> </div> </div>	[4]
i. How changes in the interest rate affect the future value of the annuity?	[2.5]

ii. How would the future value change with each increase in the investment duration?	[1.5]
<p>Ans: Given that: $a = Nu500$, $i = 0.06$ and $n = 10$ yrs</p> $FV_1 = \frac{a}{i} [(1+i)^n - 1]$ $FV_1 = \frac{500}{0.06} [(1+0.06)^{10} - 1] = 8333.3 [(1.06)^{10} - 1]$ $\Rightarrow 8333.3 \times 0.7908 = 6589.97$ $FV_2 = \frac{500}{0.08} [(1+0.08)^{10} - 1] = 6250 [(1.08)^{10} - 1]$ $\Rightarrow 6250 \times 1.1589 = 7243.13$ <p>i. An increase in the interest rate from 6% to 8% leads to a higher accumulation of the future value, rising from Nu 6,589.97 to Nu 7,243.13.</p> $FV_3 = \frac{500}{0.06} [(1+0.06)^{15} - 1] = 8333.3 [(1.06)^{15} - 1]$ $\Rightarrow 8333.3 \times 1.3966 = 11,638.3$ <p>ii. The future value of the annuity increases significantly with each extension of the investment duration, growing from Nu 6,589.97 to Nu 11,638.30.</p>	<p>0.5</p> <p>0.5</p> <p>0.5</p> <p>0.5</p> <p>0.5</p> <p>0.5</p> <p>0.5</p>
Question 13	
a) Determine the probability that either A or B occurs if the two events A and B are such that $P(A) = 0.8$, $P(B) = 0.3$ and $P(A/B) = 0.6$.	[2]
<p>Ans: $P(A \cap B) = P(B) \times P(A/B)$</p> $= 0.3 \times 0.6$ $= 0.18$ <p>Then, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$</p> $= 0.8 + 0.3 - 0.18$ $= 1.1 - 0.18$ $= 0.92$	<p>0.5</p> <p>0.5</p> <p>0.5</p> <p>0.5</p>

b) Mr. Sonam gave RCSC preliminary examinations coaching to graduate students. He grouped them in pairs so that students in a pair have nearly equal scores from first mock test. In each pair one student was taught by method I and the other by method II. The marks obtained by them are as follows: (65, 67), (66, 68), (67, 65), (67, 68), (68, 72), (69, 72), (70, 69), (72, 71). Find the rank correlation coefficient and interpret the result. [5]

Ans:

Method(I)	Method (II)	(R_1)	(R_2)	$D = R_1 - R_2$	D^2
65	67	8	7	1	1
66	68	7	5.5	1.5	2.25
67	65	5.5	8	-2.5	6.25
67	68	5.5	5.5	0	0
68	72	4	1.5	2.5	6.25
69	72	3	1.5	1.5	2.25
70	69	2	4	-2	4
72	71	1	3	-2	4
					$\sum D^2 = 26$

0.5

0.5

0.5

In the method I series, 67 occurs twice and its rank is $\frac{5+6}{2} = 5.5$

In the method II series, 72 occurs twice and its rank is $\frac{1+2}{2} = 1.5$

The correction factor for the method I series = $\frac{m^3 - m}{12} = \frac{2^3 - 2}{12} = 0.5$

The correction factor for method II series = $\frac{1}{12}(2^3 - 2) + \frac{1}{12}(2^3 - 2) = 0.5 + 0.5 = 1$

Total correction factor = $0.5 + 1 = 1.5$

0.5

0.5

0.5

$R = 1 - \frac{6[\sum D^2 + 1.5]}{n(n^2 - 1)}$	}	0.5
$\Rightarrow 1 - \frac{6[26 + 1.5]}{8(8^2 - 1)} = 1 - \frac{6 \times 27.5}{504} = \frac{339}{504} = 0.67$	}	1
<p>The correlation between the test scores obtained by using method I and method II is positive moderate correlation.</p>		}