SECTION A (30 MARKS)

ANSWER **ALL** QUESTIONS

a) For each question, there are FOUR responses: A, B, C, and D. Choose the corresponding letter of your response and CIRCLE it neatly. NO score will be awarded if you circle more than ONE letter. **[30]**

i. For every matrix $A_{m\times n}$, there exist a matrix B such that $A_{m\times n} + B = 0$. Then matrix B is [2]

A $A_{n\times m}$.

B $A_{m\times n}$.

C $-A_{n\times m}$.

(D) $\boldsymbol{-A_{m\times n}}$.

Ans: D

$A_{m\times n} + B = 0$

$A_{m\times n} + (-A_{m\times n}) = 0$

$B = -A_{m\times n}$[Additive Inverse Law]

Criteria	**Marks**
Circles the correct option	**2**
Circles more than ONE alternative	**0**
Circles none of the alternatives	**0**

ii. What is $\frac{dy}{dx}$, if $x = \cos t$ and $y = e^{\log_e \sin t}$? [2]

A $\cot t$

B $\tan t$

(C) $\boldsymbol{-\cot t}$

D $-\tan t$

Ans: C

$y = e^{\log_e \sin t} = \sin t \quad and \quad x = \cos t$

$\frac{dy}{dt} = \cos t, \qquad \frac{dx}{dt} = -\sin t$

$\frac{dy}{dx} = \frac{\cos t}{-\sin t} = -\cot t$

iii. Find the principal value of $\sin^{-1}\left(\cos\frac{2\pi}{3}\right)+\cos^{-1}\left(\sin\frac{3\pi}{4}\right)$. A $\frac{\pi}{4}$ B $\frac{5\pi}{12}$ (C) $\boldsymbol{\frac{\pi}{12}}$ D $-\frac{\pi}{6}$	[2]
Ans: C $\sin^{-1}(\cos 120°)+\cos^{-1}(\sin 135°)$ $\sin^{-1}(\cos(90+30))+\cos^{-1}(\sin(90+45))$ $\sin^{-1}(-\sin 30)+\cos^{-1}(\cos 45)$ $\sin^{-1}\left(-\frac{1}{2}\right)+\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ $-\sin^{-1}\left(\frac{1}{2}\right)+\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$ $-\frac{\pi}{6}+\frac{\pi}{4}=\frac{\pi}{12}$	
iv. A class of 36 students is to be divided equally among the teachers teaching English, Dzongkha, Physics, Chemistry, Biology and Mathematics. In how many ways can these students be distributed among the subject groups for them to attend remedial classes? A $\frac{(36)!}{6!}$ (B) $\boldsymbol{\frac{(36)!}{(6!)^6}}$ C $\frac{(36)!}{6(6!)^6}$ D $\frac{(36)!}{6!(6!)^6}$	[2]
Ans: B	

$\frac{(36)!}{6!(6!)^6} \times 6! = \frac{(36)!}{(6!)^6}$	
v. Evaluate: $\int_{-2}^{2} \log\left(\frac{1+x}{1-x}\right) dx$ (A) 0 B 1 C 2 D 3	[2]
Ans: A $\int_{-2}^{2} \log\left(\frac{1+x}{1-x}\right) dx$ $f(x) = \log\left(\frac{1+x}{1-x}\right)$ $f(-x) = \log\left(\frac{1-x}{1+x}\right) = -\log\left(\frac{1+x}{1-x}\right) =$ odd function $\therefore \int_{-2}^{2} \log\left(\frac{1+x}{1-x}\right) dx = 0$	
vi. Which of the following differential equations are linear? I $\frac{dy}{dx} = \frac{x^4 + 2xy + 1}{1 - x^2}$ II $\sin x \frac{dy}{dx} = y - \cos x$ III $y \frac{dy}{dx} = e^{\cos x} + \log x$ IV $\frac{d^2y}{dx^2} + 2xy - 5x = 0$ A I & III B III & IV C I, II & III (D) I, II & IV	[2]
Ans: D	

I: $\frac{dy}{dx} = \frac{x^4 + 2xy + 1}{1 - x^2}$,

the degree of differential coefficient and dependent variable is 1 and hence linear.

II: $\sin x \frac{dy}{dx} = y - \cos x$,

the degree of differential coefficient and dependent variable is 1 and hence linear.

III: $y \frac{dy}{dx} = e^{\cos x} + \log x$,

the differential coefficient and dependent variable are multiplied and hence non-linear.

IV: $\frac{d^2 y}{dx^2} + 2xy - 5x = 0$,

the order of the differential coefficient is 2 but degree is 1 and hence linear.

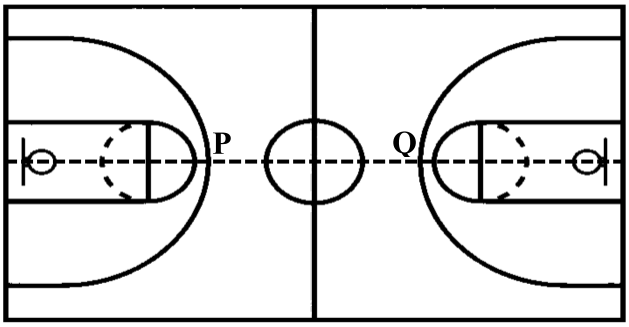
vii. If the equation of a 3-pointer curve of a basketball court is $9x^2 - 16y^2 = 144$, what is the horizontal distance between the points P and Q marked in the figure below? [2]

A 6 units

(B) 8 units

C 9 units

D 16 units



Ans: B

$9x^2 - 16y^2 = 144$

$\frac{9x^2}{144} - \frac{16y^2}{144} = \frac{144}{144}$

$\frac{x^2}{16} - \frac{y^2}{9} = 1$

$a^2 = 16,\ b^2 = 9$

The distance PQ is the distance between two vertices

$= 2a$

$= 2 \times 4 = 8$ units

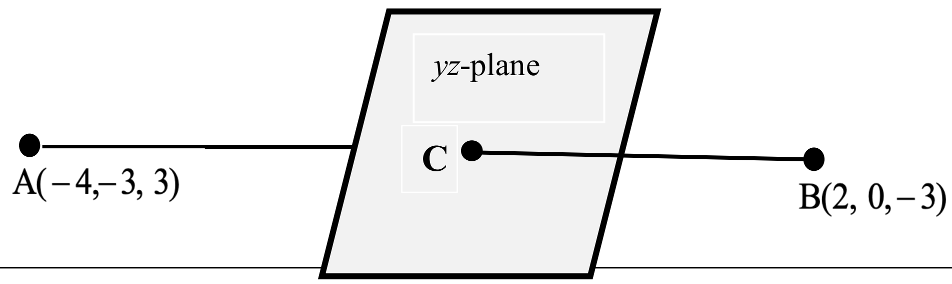
viii. If $z_1 = 2 + 3i$ and $z_2 = 1 - 4i$, find the amplitude of $z_1 \times z_2$. [2]

A -70.35^0

(B) $\mathbf{-19.65^0}$

C 19.65^0

D 70.35^0

Ans: B $z_1 = 2+3i$ $z_2 = 1-4i$ $z_1 \times z_2 = (2+3i)(1-4i)$ $= (2+12)-5i$ $= 14-5i$ $\text{Amp}(z_1 \times z_2) = -\tan^{-1}\left\|\frac{5}{14}\right\| = -19.65°$	
ix. The distance covered by a beetle crawling on the ground in time *t* seconds is given by $s = 2(t-2)^3 + 5t$. For the velocity to be minimum, what should be the time taken to cover the distance? A $\frac{1}{6}$ sec B $\frac{1}{2}$ sec C 1 sec (D) **2 sec**	[2]
Ans: D $s = 2(t-2)^3 + 5t$ $\text{Velocity(v)} = \frac{ds}{dt} = 6(t-2)^2 + 5$ $\frac{dv}{dt} = \frac{d^2s}{dt^2} = 12(t-2)$ For maximum and minimum value $\frac{d^2s}{dt^2} = 0$ $\therefore\ 12(t-2) = 0, \quad t = 2$ For the minimum velocity $\frac{d^3s}{dt^3} > 0$ $\frac{d^3s}{dt^3} = 12$, which is greater than 0. $\therefore$ Velocity is minimum at $t = 2$ sec	
x. Find the coordinates of C.  	[2]

	A $(\mathbf{0, -1, -1})$ B $(-1, 0, -6)$ C $\left(0, \frac{-3}{2}, \frac{-3}{2}\right)$ D $\left(-1, \frac{3}{2}, \frac{-3}{2}\right)$	
	Ans: A Let the ratio in which the plane divides the line joining points be $k:1$ On yz plane $x = 0$, using section formula $\frac{2k + (-4)}{k+1} = 0$ $k = 2$ $y = \frac{-3}{3} = -1, z = \frac{-6+3}{3} = -1$ $\therefore$ Point of intersection is $(0, -1, -1)$	
xi.	In a fair, Karma and Sonam threw 6 identical darts at a target $10\,m$ away. The following represents the probability associated between them: Probability of Karma or Sonam not hitting the target is $\frac{1}{6}$ Probability of Karma and Sonam hitting the target is $\frac{1}{4}$ Probability of Karma not hitting the target is $\frac{1}{4}$ What is the relation between the events of hitting the target by both Karma and Sonam? A Dependent and mutually exclusive. B Independent and mutually exclusive. C Dependent and not mutually exclusive. D Independent and not mutually exclusive.	[2]
Ans: D		

Let event A be Karma hitting the target and B be Sonam hitting the target. Given that $P(\overline{A \cup B}) = \frac{1}{6}$ $P(A \cap B) = \frac{1}{4}$ $P(\overline{A}) = \frac{1}{4}$ If A and B are independent, $P(A \cap B) = P(A).P(B)$ $P(A) = 1 - \frac{1}{4} = \frac{3}{4}$ $P(A \cup B) = 1 - \frac{1}{6} = \frac{5}{6}$ We have $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $\frac{5}{6} = \frac{3}{4} + P(B) - \frac{1}{4}$ $P(B) = \frac{1}{3}$ $P(A \cap B) = P(A).P(B)$ $\frac{1}{4} = \frac{3}{4} \times \frac{1}{3}$ $\frac{1}{4} = \frac{1}{4}$, hence independent Since $P(A \cap B) = \frac{1}{4}$, it is not mutually exclusive $\therefore$ Events A and B are independent and not mutually exclusive.	
xii. What are the angles made by the normal to the plane $x - 2y + 2z - 4 = 0$ with the axes? A $Cos^{-1}\left(\frac{1}{3}\right), Cos^{-1}\left(\frac{2}{3}\right), Cos^{-1}\left(\frac{2}{3}\right)$ B $Cos^{-1}\left(\frac{1}{9}\right), Cos^{-1}\left(-\frac{2}{9}\right), Cos^{-1}\left(\frac{2}{9}\right)$ (C) $\mathbf{Cos^{-1}\left(\frac{1}{3}\right), Cos^{-1}\left(-\frac{2}{3}\right), Cos^{-1}\left(\frac{2}{3}\right)}$ D $Cos^{-1}\left(\frac{1}{\sqrt{3}}\right), Cos^{-1}\left(\frac{2}{\sqrt{3}}\right), Cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$	[2]
Ans: C	

$$x - 2y + 2z - 4 = 0$$
$$a = 1, b = -2, c = 2$$
$$\cos\alpha = \frac{a}{\sqrt{a^2 + b^2 + c^2}} = \frac{1}{3}$$
$$\alpha = \cos^{-1}\left(\frac{1}{3}\right)$$
$$\beta = \cos^{-1}\left(\frac{-2}{3}\right)$$
$$\gamma = \cos^{-1}\left(\frac{2}{3}\right)$$

xiii. Two judges were appointed to grade six areas in SUPW. Find x if the difference between the assigned ranks were as given below. [2]

Class	A	B	C	D	E	F
Rank difference	–1	x	2.5	3.5	–2.5	–1.5

A 2.5

B 1

(C) **–1**

D –1.5

Ans: C

$$-1 + x + 2.5 + 3.5 + (-2.5) + (-1.5) = 0$$
$$x = -1$$

xiv. Athlete Mike Powell from the United States has a World record of 9 m in men's long jump. He initially accelerates for a few meters and then takes a jump once he gains momentum. If the locus of his jump is given by $y = 9x - x^2$, what is the area under the curve and the horizontal ground? [2]

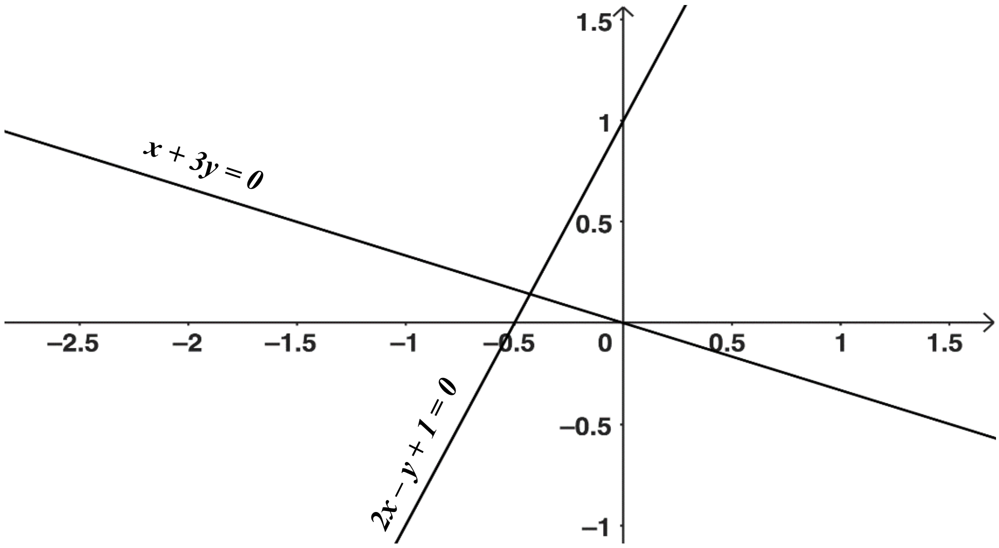
(A) **121.5 m^2**

B 243.0 m^2

C 364.5 m^2

D 607.5 m^2

Ans: A

The function of a given curve is $y = 9x - x^2$ Area under a curve from 0 to 9 $=\int_0^9 9x - x^2 dx = \left[\frac{9x^2}{2} - \frac{x^3}{3}\right]_0^9$ $= \left[\frac{9(9)^2}{2} - \frac{(9)^3}{3}\right] - 0$ $= 121.5\, m^2$	
xv. Find the equation of a pair of lines passing through $(-2,1)$ and parallel to the given lines.  A $2x^2 - 5xy + 3y^2 + 3x + 16y - 5 = 0$ (B) $\mathbf{2x^2 + 5xy - 3y^2 + 3x + 16y - 5 = 0}$ C $2x^2 - 7xy + 3y^2 + 3x + 14y + 5 = 0$ D $2x^2 + 7xy - 3y^2 + 3x + 14y + 5 = 0$	[2]
Ans: B $(x+3y)(2x-y+1) = 2x^2 - xy + x + 6xy - 3y^2 + 3y$ $= 2x^2 + 5xy - 3y^2 + x + 3y = 0$.....................$(i)$ Equation of lines parallel to (i) and passing through (-2,1) is given by $2(x+2)^2 + 5(x+2)(y-1) - 3(y-1)^2 = 0$ $2x^2 + 5xy - 3y^2 + 3x + 16y - 5 = 0$	

SECTION B (70 MARKS)

ANSWER ANY **TEN** QUESTIONS

Question 2

a) The table below shows the distribution in the consumption of different types of energy by % for cooking between rural and urban areas in Wangdue Dzongkhag as per Population and Housing Census of Bhutan 2017. Evaluate the coefficient of correlation of the main type of energy consumption between rural and urban area. [3]

Area	Electricity	Kerosene	Firewood	Bio-Gas	LPG
Urban	98.6	0.3	0.3	1.8	92.4
Rural	95.8	1.3	12.9	1.3	79.8

Urban (x)	Rural (y)	x^2	y^2	xy	
98.6	95.8	9721.96	9177.64	9445.88	
0.3	1.3	0.09	1.69	0.39	
0.3	12.9	0.09	166.41	3.87	
1.8	1.3	3.24	1.69	2.34	
92.4	79.8	8537.76	6368.04	7373.52	

Table with correct values……………………………………………..[1]

$$\sum x = 193.4, \sum y = 191.1, \sum x^2 = 18263.14, \sum y^2 = 15715.47, \sum xy = 16826$$ ....[1]

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - \left(\sum x\right)^2}\sqrt{n\sum y^2 - \left(\sum y\right)^2}}$$

$$r = \frac{5(16826) - (193.4)(191.1)}{\sqrt{5(18263.14) - (193.4)^2}\sqrt{5(15715.47) - (191.1)^2}}$$ ....[1]

$$r = 0.991$$

b) Kinley, Dema and Laxmi have a total sum of Nu 284. Dema has one third what Kinley has, and Laxmi has Nu 24 more than Dema. Using matrix method, calculate how much money each has. [4]

Let x, y and z be an amount of money kinley, Dema and Laxmi have for the contributions

Given that $x+y+z=284$

Also given that $y=\frac{1}{3}x \Rightarrow x-3y=0$

and $z=y+25 \Rightarrow -y+z=25$ ....[1]

From the three equations above;

$$A=\begin{bmatrix}1 & 1 & 1\\ 1 & -3 & 0\\ 0 & -1 & 1\end{bmatrix} \ and \ B=\begin{bmatrix}284\\ 0\\ 25\end{bmatrix}$$

$|A|=1(-3)-1(2)=-3-2=-5$ ....[0.5]

$$AdjA=\begin{bmatrix}-3 & -2 & 3\\ -1 & 1 & 1\\ -1 & 1 & -4\end{bmatrix}$$ ....[1]

$$X=\frac{1}{-5}\begin{bmatrix}-3 & -2 & 3\\ -1 & 1 & 1\\ -1 & 1 & -4\end{bmatrix}\begin{bmatrix}284\\ 0\\ 25\end{bmatrix}$$

$$X=\frac{1}{-5}\begin{bmatrix}-280\\ -260\\ -380\end{bmatrix}=\begin{bmatrix}156\\ 52\\ 76\end{bmatrix}$$ ....[1]

$\therefore x=156,\ y=52\ and\ z=76.$

Kinley, Dema and Laxmi have Nu 156, Nu 52 and Nu 76 respectively ....[0.5]

Question 3 [3]

a) Find the equation for the bisector of angles of the pair of lines perpendicular to the lines $-2x^2+5xy+y^2=0.$

Equation of line perpendicular to $-2x^2+5xy+y^2=0$

is $x^2-5xy-2y^2=0$...............................(i) [1]

$a=1,\ b=-2,\ h=\frac{-5}{2}$

Equation of bisector of angles of (i) is

$$\frac{x^2-y^2}{a-b}=\frac{xy}{h}$$

$$\frac{x^2-y^2}{1-(-2)}=\frac{xy}{-5/2}$$ ....[1]

$$\frac{x^2-y^2}{3}=\frac{-2xy}{5}$$

$$5x^2-5y^2+6xy=0$$ ....[1]

b)	Kinley constructed an open tank with a square base of side x m and height h m to store $4m^3$ of water. He wants to paint the interior walls of the tank. What would be the minimum cost incurred if a litre of paint cost Nu 350 which can paint an area of 3 m^2,?	[4]

$$\left.\begin{array}{l}\text{Volume(v)}=4m^3\\ x^2h=4\Rightarrow h=\dfrac{4}{x^2}\end{array}\right\}\text{.......................................[0.5]}$$

$$\left.\begin{aligned}\text{Surface area(A)}&=x^2+4hx\\ &=x^2+4\left(\frac{4}{x^2}\right)x\\ &=x^2+\frac{16}{x}\end{aligned}\right\}\text{..............................[0.5]}$$

$$\left.\frac{dA}{dx}=2x-\frac{16}{x^2}=\frac{2x^3-16}{x^2}\right\}\text{..............................[0.5]}$$

$$\left.\begin{array}{l}\text{When }\dfrac{dA}{dx}=0,\dfrac{2x^3-16}{x^2}=0\Rightarrow 2x^3-16=0\\ 2x^3-16=0\Rightarrow 2x^3=16\Rightarrow x^3=8\\ \therefore x=2\end{array}\right\}\text{..............[0.5]}$$

$$\left.\frac{d^2A}{dx^2}=2+\frac{32}{x^3}=2+\frac{32}{8}=6>0\right\}\text{..............................[0.5]}$$

$$\left.\begin{aligned}h&=\frac{4}{2^2}=1\\ \text{Surface Area}&=x^2+4xh\\ &=4+(4\times2\times1)=12m^2\end{aligned}\right\}\text{..........................[0.5]}$$

$$\left.\begin{array}{l}3m^2=1L\\ 12m^2=\dfrac{1}{3}\times12=4L\end{array}\right\}\text{.......................................[0.5]}$$

$$\left.\begin{array}{l}1L=\text{Nu}\,350\\ 4L=4\times350=\text{Nu}\,1400\end{array}\right\}\text{..............................[0.5]}$$

Question 4

a)	To be awarded a Pass Certificate in the class XII examination, a student must pass in English and Dzongkha, and any other two subjects. A class XII Science student estimated his chance of passing in English as $\frac{4}{5}$, Physics as $\frac{5}{6}$, Chemistry as $\frac{2}{3}$ and Mathematics as $\frac{3}{4}$. What is the probability that he would be awarded a Pass Certificate on the condition that he received a pass mark in Dzongkha?	[3]

Let E, P, C and M be the events of passing in English, Physics, Chemistry and Mathematics respectively. And $\overline{E}, \overline{P}, \overline{C}, and\ \overline{M}$ events of failing in English, Physics, Chemistry and Mathematics respectively. $P(E)=\frac{4}{5}$ $\left.\begin{array}{l} P(P)=\frac{5}{6} \text{ and } P(\overline{P})=1-\frac{5}{6}=\frac{1}{6} \\ P(C)=\frac{2}{3} \text{ and } P(\overline{C})=1-\frac{2}{3}=\frac{1}{3} \\ P(M)=\frac{3}{4} \text{ and } P(\overline{M})=1-\frac{3}{4}=\frac{1}{4} \end{array}\right\}$..[1] He will pass the exam under following conditions, $=\left.\begin{array}{l} P(E)\times P(P)\times P(C)\times P(M)+P(E)\times P(\overline{P})\times P(C)\times P(M)+ \\ P(E)\times P(P)\times P(\overline{C})\times P(M)+P(E)\times P(C)\times P(C)\times P(\overline{M}) \end{array}\right\}$.............[1] $\left.\begin{array}{l} =\left(\frac{4}{5}\times\frac{5}{6}\times\frac{2}{3}\times\frac{3}{4}\right)+\left(\frac{4}{5}\times\frac{1}{6}\times\frac{2}{3}\times\frac{3}{4}\right)+\left(\frac{4}{5}\times\frac{5}{6}\times\frac{1}{3}\times\frac{3}{4}\right)+\left(\frac{4}{5}\times\frac{5}{6}\times\frac{2}{3}\times\frac{1}{4}\right) \\ =\frac{1}{3}+\frac{1}{15}+\frac{1}{6}+\frac{1}{9}=0.744 \end{array}\right\}$...............[1]	
b) Show that the equation $x^2-4xy+4y^2+4x-8y+3=0$ represents a pair of parallel lines and find the distance between them.	[4]
Ans $x^2-4xy+4y^2+4x-8y+3=0$ $x^2+(4-4y)x+(4y^2-8y+3)=0$...[0.5] $\left.\begin{array}{l} x=\frac{-4+4y\pm\sqrt{(4-4y)^2-4(4y^2-8y+3)}}{2} \\ x=\frac{-4+4y\pm\sqrt{16-32y+16y^2-16y^2+32y-12}}{2} \\ x=\frac{-4+4y\pm 2}{2} \end{array}\right\}$.................[1] $\left.\begin{array}{l} 2x=-6+4y \Rightarrow x-2y=-3............(i) \\ 2x=-2+4y \Rightarrow x-2y=-1............(ii) \end{array}\right\}$................................[0.5] Slope of equation (i) $m_1=2$ Slope of equation (ii) $m_2=2$ ....[0.5] $\therefore$ They are parallel line. Let $(x,0)$ be point on equation (ii) $\Rightarrow x=-1$ ....[0.5] $\therefore$ $(-1,0)$ is a point on equation (ii) Distance between two lines$=\left\|\frac{-1-0+3}{\sqrt{1+4}}\right\|=\frac{2}{\sqrt{5}}$ *units*....................[1]	

Question 5	
a) A school has 7 periods of 40 minutes each in a day. Commerce stream has 6 different subjects. In how many ways can we organize these subjects such that each subject is allocated at least one period in a day?	[3]
Arrangement of 6 subjects in 7 periods can be done in ${}^7P_6$ ways............................[1] Selection of 6 subjects for remaining 1 period can be done in ${}^6C_1$ ways.................[1] Organising 6 subjects in 7 periods can be done in ${}^7P_6 \times {}^6C_1$ ways Since one subject is allocated in two periods, $\Rightarrow \frac{{}^7P_6 \times {}^6C_1}{2!} = \frac{5040 \times 6}{2} = 15120$ ways..........[1]	
b) Compute eccentricity, foci, equation of directrices, length of latus rectum and length of axes of conic section $8x^2 + 6y^2 = 96$.	[4]
$8x^2 + 6y^2 = 96$, $\frac{8x^2}{96} + \frac{6y^2}{96} = 1$, $\frac{x^2}{12} + \frac{y^2}{16} = 1$ ....[0.5] $a^2 = 16, b^2 = 12$ ....[0.5] Eccentricity $e = \sqrt{1 - \frac{b^2}{a^2}} \Rightarrow \sqrt{1 - \frac{12}{16}} = \sqrt{\frac{4}{16}} = \frac{1}{2}$ ....[0.5] $Foci = (0, \pm ae) = \left(0, \pm 4 \times \frac{1}{2}\right) = (0, \pm 2)$ ....[0.5] $Equation\ of\ directrices\ y = \pm \frac{a}{e} = \pm \frac{4}{\frac{1}{2}} = \pm 8$ ....[0.5] $Length\ of\ latus\ rectum = \frac{2b^2}{a} = \frac{2 \times 12}{4} = 6$ ....[0.5] $Length\ of\ axes = 2a = 2 \times 4 = 8$ and $2b = 2 \times \left(2\sqrt{3}\right) = 4\sqrt{3}$ ....[1]	

Question 6 [3]

a) If $A=\begin{bmatrix}2 & x & -1\\0 & 1 & y\\2 & -1 & 3\end{bmatrix}$, $B=\begin{bmatrix}4 & 3 & 1\\y & -1 & 1\\2 & x & -1\end{bmatrix}$ and $3A'+B=\begin{bmatrix}10 & 3 & 7\\2 & 2 & -2\\-1 & 2 & 8\end{bmatrix}$. Find the value of x and y.

Ans:

$$A'=\begin{bmatrix}2 & 0 & 2\\x & 1 & -1\\-1 & y & 3\end{bmatrix} \text{.................[0.5]}$$

$$3A'+B=3\begin{bmatrix}2 & 0 & 2\\x & 1 & -1\\-1 & y & 3\end{bmatrix}+\begin{bmatrix}4 & 3 & 1\\y & -1 & 1\\2 & x & -1\end{bmatrix}=\begin{bmatrix}10 & 3 & 7\\3x+y & 2 & -2\\-1 & 3y+x & 8\end{bmatrix} \text{...[0.5]}$$

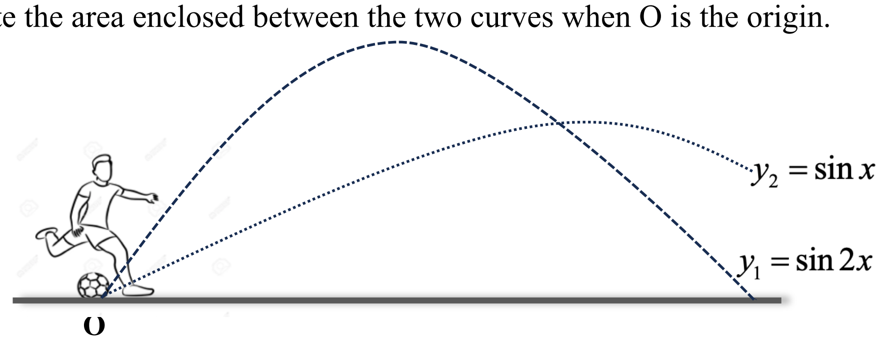
$$\begin{bmatrix}10 & 3 & 7\\3x+y & 2 & -2\\-1 & 3y+x & 8\end{bmatrix}=\begin{bmatrix}10 & 3 & 7\\2 & 2 & -2\\-1 & 2 & 8\end{bmatrix}$$

$$3x+y=2$$
$$x+3y=2\times 3 \quad \text{.................[0.5]}$$

$$3x+y=2$$
$$-3x+9y=-6$$
$$-8y=-4 \quad \text{.................[1]}$$
$$y=\frac{1}{2}$$

$$x=2-\left(3\times\frac{1}{2}\right)$$
$$x=\frac{1}{2} \quad \text{.................[0.5]}$$

b) Chencho kicked a ball twice and the path traced by the ball is shown in the figure below. Calculate the area enclosed between the two curves when O is the origin. [4]



Ans:

Point of intersection between curves y_1 *and* y_2

$\sin 2x = \sin x \Rightarrow 2\sin x \cos x = \sin x$

$\cos x = \frac{1}{2}$

$\therefore$ two curves intersect at $x = \cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$ ....[1]

Area enclosed by two curves $= \int_0^{\frac{\pi}{3}} (y_1 - y_2)dx$ ....[0.5]

$$= \int_0^{\frac{\pi}{3}} (\sin 2x - \sin x)dx$$

$$= \int_0^{\frac{\pi}{3}} \sin 2x dx - \int_0^{\frac{\pi}{3}} \sin x dx$$

$$= \left[\frac{-\cos 2x}{2}\right]_0^{\frac{\pi}{3}} + [\cos x]_0^{\frac{\pi}{3}}$$ ....[1]

$$= -\frac{1}{2}\left[\cos 2\frac{\pi}{3} - \cos 0\right] + \left[\cos\frac{\pi}{3} - \cos 0\right]$$

$$= -\frac{1}{2}\left[\cos\frac{2\pi}{3} - \cos 0\right] + \left[\cos\frac{\pi}{3} - \cos 0\right] =$$

$$= -\frac{1}{2}[-0.5 - 1] + [0.5 - 1]$$ ....[1]

$$= \frac{1}{4}\text{sq.unit}$$ ....[0.5]

Question 7 [3]

a) The equation of two regression lines is $2y = 5x - 8$ and $3x - 4y + 12 = 0$. Evaluate the coefficient of correlation, regression coefficients b_{yx} and b_{xy}, mean of x and y and the value of x when $y = 11$.

Ans

$x\,on\,y$ $\qquad\qquad$ $y\,on\,x$

$2y = 5x - 8$

$2y - 5x + 8 = 0$ $\qquad\qquad$ $3x - 4y + 12 = 0$

$b_{xy} = \dfrac{-coefficient\ of\ y}{coefficient\ of\ x} = \dfrac{-2}{-5} = \dfrac{2}{5}\Big\}$..........[0.5] $\quad b_{yx} = \dfrac{-coefficient\ of\ y}{coefficient\ of\ x} = \dfrac{-3}{-4} = \dfrac{3}{4}\Big\}$.....[0.5]

$r = \pm\sqrt{b_{xy} \times b_{yx}} = \pm\sqrt{\dfrac{2}{5} \times \dfrac{3}{4}} = \pm\sqrt{\dfrac{3}{10}}$

Q b_{xy} and b_{yx} are positive r = $\sqrt{\dfrac{3}{10}}$ ....[0.5]

$$-5x + 2y + 8 = 0$$
$$3x - 4y + 12 = 0$$
$$\frac{x}{24+32} = \frac{y}{24+60} = \frac{1}{20-60}$$
$$x = \frac{56}{14} = 4, \quad y = \frac{84}{14} = 6$$
......[1]

Point $\left(\bar{x}, \bar{y}\right) = (4, 6)$

$x\ on\ y: 2y - 5x + 8 = 0$

$2(11) - 5x + 8 = 0$

$x = 6$

...........................[0.5]

b) Verify if $y = -x\cos x + 2\sin x + c_1 x + c_2$ is the general solution for the differential equation $\dfrac{d^2y}{dx^2} = x\cos x$. Next, find the particular solution of the differential equation if $\dfrac{dy}{dx} = -1, y = 1$ when $x = 0$. [4]

Ans:

$y = -x\cos x + 2\sin x + c_1 x + c_2$ ....(i)

Differentiate $w.r.t\ x$

$$\left.\begin{aligned}\frac{dy}{dx} &= -x(-\sin x) + \cos x(-1) + 2\cos x + c_1\\ &= x\sin x - \cos x + 2\cos x + c_1\\ \frac{dy}{dx} &= x\sin x + \cos x + c_1 \text{................}(ii)\end{aligned}\right\}\text{..[1]}$$

$$\left.\begin{aligned}\frac{d^2y}{dx^2} &= x\cos x + \sin x(1) - \sin x\\ &= x\cos x + \sin x - \sin x\\ &= x\cos x\end{aligned}\right\}\text{..[1]}$$

$\therefore$ The given function is the solution to the given D.E$\}$.........................[0.5]

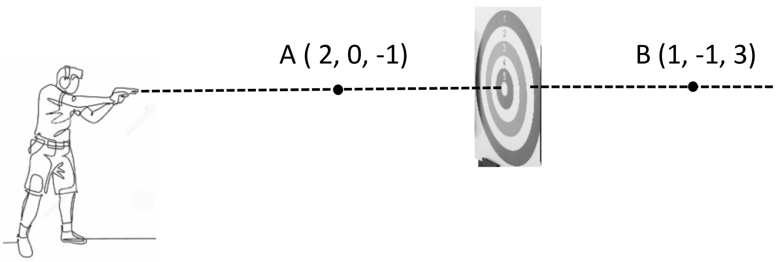
$$\left.\begin{aligned}&\text{Substituting } \frac{dy}{dx} = -1 \text{ and } x = 0 \ in\ (ii)\\ &-1 = 0\sin 0 + \cos 0 + c_1\\ &-1 = 1 + c_1 \Rightarrow c_1 = -2\end{aligned}\right\}\text{...[0.5]}$$

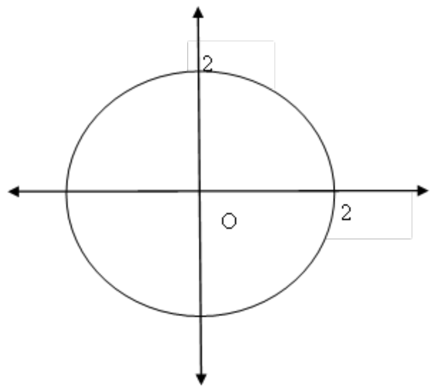
$$\left.\begin{aligned}&\text{Substituting } x = 0 \text{ and } y = 1\ in\ (i)\\ &1 = -0\cos 0 + 2\sin 0 + c_1(0) + c_2 \Rightarrow c_2 = 1\end{aligned}\right\}\text{..[0.5]}$$

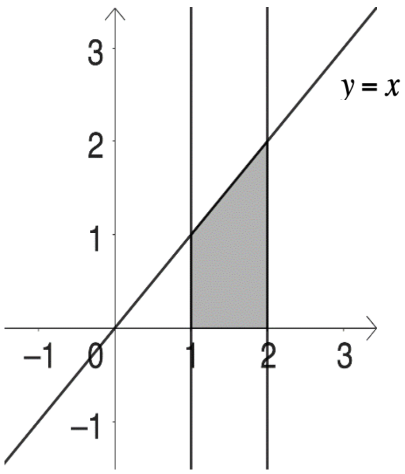
$\therefore\ y = -x\cos x + 2\sin x - 2x + 1$ is the required particular solution$\}$......[0.5]

Question 8 [3]

a) The man is shooting in an indoor firing range. The path of the bullet is perpendicular to the plane of the target as shown below. If $P(2,-3,1)$ is a point on the target, find the equation of the plane. Also, find the equation of line AB.



Ans: Equation of a plane passing through $(2,\ -3,\ 1)$ is; $a(x-2)+b(y+3)+c(z-1)=0$..........................(i) } .... [0.5] Direction ratios of line AB $=(1,\ 1,-4)$ }......................................[0.5] $\because$ AB is perpendicular to plane, AB is normal to the plane. $\therefore (i)$ becomes $1(x-2)+1(y+3)-4(z-1)=0$ $x-2+y+3-4z+4=0$ $x+y-4z+5=0$ } .....[1] Equation of line AB Using point $A(2,0,-1)$ Using point $B(1,\ -1,\ 2)$ $\frac{x-2}{1}=\frac{y-0}{1}=\frac{z+1}{-4}$ *OR* $\frac{x-1}{1}=\frac{y+1}{1}=\frac{z-3}{-4}$ } ....[1]	
b) Illustrate that the locus of $\text{Re}(zi)^2+2x^2=4$, if $z=x+yi$ is a complex number.	[4]
Ans: $(zi)^2=[(x+yi)i]^2$ $=(xi-y)^2=-x^2-2xyi+y^2$ $=(y^2-x^2)-2xyi$ } ....[1] $\text{Re}(zi)^2=y^2-x^2$ }.......................................[0.5] $\text{Re}(zi)^2+2x^2=4$ $\Rightarrow y^2-x^2+2x^2=4$ $x^2+y^2=4$ } ....[0.5] It represents points ON the circle with; Centre: $(0,0)$ and $r=\sqrt{4}=2$ } ....[1]  	
Question 9 a) Prove that $9x^2-12xy\cos\theta+4y^2+36\cos^2\theta-36=0$, if $\cos^{-1}\left(\frac{x}{2}\right)+\cos^{-1}\left(\frac{y}{3}\right)=\theta$.	[3]

$\cos^{-1}\left(\frac{x}{2}\right)+\cos^{-1}\left(\frac{y}{3}\right)=\theta$ $\cos^{-1}\left(\frac{x}{2}\frac{y}{3}-\sqrt{1-\frac{x^2}{4}}\sqrt{1-\frac{y^2}{9}}\right)=\theta$ $\frac{x}{2}\frac{y}{3}-\sqrt{1-\frac{x^2}{4}}\sqrt{1-\frac{y^2}{9}}=\cos\theta$ ....[0.5] $\frac{xy}{6}-\sqrt{\frac{4-x^2}{4}}\sqrt{\frac{9-y^2}{9}}=\cos\theta$ $\frac{xy}{6}-\frac{\sqrt{4-x^2}}{2}\frac{\sqrt{9-y^2}}{3}=\cos\theta$ ....[0.5] $\frac{xy}{6}-\frac{\sqrt{4-x^2}\sqrt{9-y^2}}{6}=\cos\theta$ $xy-\sqrt{4-x^2}\sqrt{9-y^2}=6\cos\theta$ $\sqrt{4-x^2}\sqrt{9-y^2}=xy-6\cos\theta$ ....[0.5] sq.both sides $(4-x^2)(9-y^2)=x^2y^2-12xy\cos\theta+36\cos^2\theta$ ....[0.5] $36-4y^2-9x^2+x^2y^2=x^2y^2-12xy\cos\theta+36\cos^2\theta$ $36-4y^2-9x^2=-12xy\cos\theta+36\cos^2\theta$ [1] $9x^2+4y^2-12xy\cos\theta+36\cos^2\theta-36=0$, hence proved.	
b) A man going for a trek walks with a uniform velocity. Define a function which will describe the distance covered with respect to time. Graph your function and identify any two points on the x-axis. Rotate the area bounded by the graph and the line passing through the points about x-axis through four right angles and evaluate the volume of the shape so formed.	[4]
Sample answer: Since the velocity is uniform, the relationship between distance and time will be linear. So let the equation be: $y=x$ .....[1] Let $x=1$ and $x=2$ be the two points. ....[0.5] When the area enclosed is rotated, the volume of the solid generated is: ....[0.5] Volume $=\pi\int_1^2 y^2dx=\pi\int_1^2 x^2dx$ Volume $=\pi\left[\frac{x^3}{3}\right]_1^2=\pi\left(\frac{8}{3}-\frac{1}{3}\right)=\frac{7}{3}\pi$ cu.units......[1]  Graph........[1]	

Question 10	
a) Convert $\dfrac{8-6i}{(1-i)^2}$ into polar form.	[3]
$\dfrac{8-6i}{(1-i)^2}=\dfrac{8-6i}{1-1-2i}=\dfrac{2(4-3i)}{-2i}=\dfrac{(4-3i)}{-i}$ ……………………[1] $\dfrac{(4-3i)}{-i}\times\dfrac{i}{i}=4i+3$, $z=3+4i$ ……………………[0.5] Polar form of a complex number $=r(\cos\theta+i\sin\theta)$ $a=3,\ b=4$ (θ lies in *first* quadrant $\theta=\alpha$) $\tan\theta=\dfrac{b}{a}\Rightarrow\theta=\tan^{-1}\left(\dfrac{b}{a}\right)=\tan^{-1}\left(\dfrac{4}{3}\right)=53.13°$ ……………[0.5] $r=\sqrt{(3)^2+(4)^2}=\sqrt{25}=5$ ……………………[0.5] Polar form $=5(\cos 53.13°+i\sin 53.13°)$ ……………………[0.5]	
b) A house has a window in the form of a rectangle surmounted by a semicircle. If the perimeter of the window is 12 *m*, find the dimensions so as to admit maximum light into the room.	[4]
Let r be the radius of the circle, x and $2r$ be the width and length of the rectangle respectively. Perimeter $=12\ m$ $2x+2r+\pi r=12$ $2x=12-2r-\pi r$ $x=6-r-\dfrac{\pi r}{2}$ ……………………[0.5] $\text{Area}(A)=2xr+\dfrac{1}{2}\pi r^2$ ……………………[0.5] $=2r\left(6-r-\dfrac{\pi r}{2}\right)+\dfrac{1}{2}\pi r^2=12r-2r^2-\pi r^2+\dfrac{1}{2}\pi r^2$ $A=12r-2r^2-\dfrac{1}{2}\pi r^2$ ….[0.5] $\dfrac{dA}{dr}=12-4r-\pi r$ ……………………[0.5] When $\dfrac{dA}{dr}=0$ $12-4r-\pi r=0$ $r(4+\pi)=12$ $r=\dfrac{12}{4+\pi}\ m$ ……………………[0.5] $\dfrac{d^2A}{dr^2}=-4-\pi<0$, max ……………………[0.5] width $=x=6-\dfrac{12}{4+\pi}-\dfrac{\pi}{2}\left(\dfrac{12}{4+\pi}\right)$ $=\dfrac{24+6\pi-12-6\pi}{4+\pi}=\dfrac{12}{4+\pi}\ m$ ……………………[0.5] length $=2r=2\left(\dfrac{12}{4+\pi}\right)=\dfrac{24}{4+\pi}\ m$ ……………………[0.5]	

Question 11 a) If $y=\sin(\tan^{-1}x)$, prove that $\frac{1}{3x}\frac{d^2y}{dx^2}=\frac{-1}{\left(1+x^2\right)^{\frac{5}{2}}}$.		[3]

$$y=\sin(\tan^{-1}x),$$

$$y=\sin\left(\sin^{-1}\frac{x}{\sqrt{1+x^2}}\right)$$

$$\left.y=\frac{x}{\sqrt{1+x^2}}\right\}\text{..[0.5]}$$

$$\left.\frac{dy}{dx}=\frac{\sqrt{1+x^2}(1)-x\frac{1}{2\sqrt{1+x^2}}2x}{\left(\sqrt{1+x^2}\right)^2}\right\}\text{............................[0.5]}$$

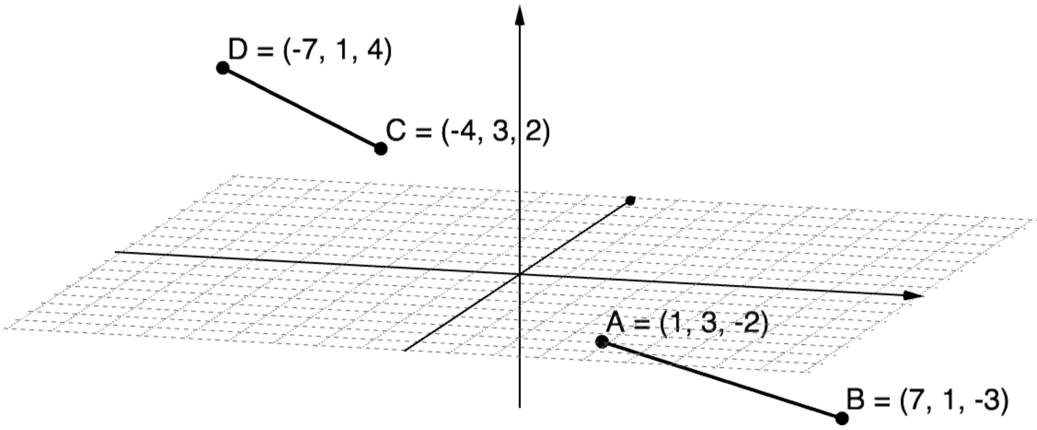
$$\left.\begin{array}{l}\frac{dy}{dx}=\left(\sqrt{1+x^2}-\frac{x^2}{\sqrt{1+x^2}}\right)\frac{1}{1+x^2}\\ \frac{dy}{dx}=\frac{1+x^2-x^2}{\left(\sqrt{1+x^2}\right)1+x^2}=\frac{1}{\left(1+x^2\right)^{\frac{3}{2}}}\end{array}\right\}\text{...........................[0.5]}$$

$$\left.\frac{d^2y}{dx^2}=-\frac{1}{\left(\left(1+x^2\right)^{\frac{3}{2}}\right)^2}\times\frac{3}{2}\left(1+x^2\right)^{\frac{1}{2}}\times 2x\right\}\text{................[0.5]}$$

$$\left.\begin{array}{l}\frac{d^2y}{dx^2}=\frac{-3x\left(1+x^2\right)^{\frac{1}{2}}}{\left(1+x^2\right)^{32}}\\ \frac{-1}{3x}\frac{d^2y}{dx^2}=\frac{1}{\left(1+x^2\right)^{\frac{5}{2}}},\text{ hence proved}\end{array}\right\}\text{.........................[1]}$$

b)	A bridge is built in the form of a parabolic arch. The highest point of the arch is 40 *ft* from the surface of water and has a span of 160 *ft*. Will a boat of 30 *ft* height be able to pass under the bridge if it is 24 *ft* from the center of the bridge? $\left(ft=\text{feet}\right)$	[4]

Equation of the parabola is $x^2 = -4ay$ }...[0.5] Let A be the heighest point be, so the coordinates are (0,40) Then the equation of the parabola is $x^2 = -4a(y-40) --- (i)$ }.............[0.5] Let B be one of the end of the arch, so the coordinates are (80,0).............[0.5] Substituting B in (i), we get $80^2 = -4a(0-40)$ $6400 = 160a \Rightarrow a = 40$ }...[0.5] So, (i) becomes $x^2 = -4(40)(y-40)$ $x^2 = -160(y-40) --- (ii)$ }...[0.5] When the ship is 24 feet from the centre, (ii) becomes $24^2 = -160(y-40)$ $576 = -160y + 6400$ $106y = 5824 \Rightarrow y = 36.4\ ft$ }..............................[1] Since the boat is 30 feet, it will be able to pass througt the bridge.}...........[0.5]	
Question 12 a) Integrate: $\int \frac{5x-1}{x^2(x-3)}\,dx$	[3]
$\int \frac{5x-1}{x^2(x-3)}\,dx$ Let $\frac{5x-1}{x^2(x-3)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-3}$ $5x-1 = Ax(x-3) + B(x-3) + Cx^2$ }......................[0.5] When $x = 3$, $14 = 9C \Rightarrow C = \frac{14}{9}$ When $x = 0$ $-1 = -3B \Rightarrow B = \frac{1}{3}$ }.... [1] $5x-1 = Ax^2 - 3Ax + Bx - 3B + Cx^2$ Equate coeff.of x^2 $0 = A + C$ $A = -C \Rightarrow A = -\frac{14}{9}$ }..................................[0.5] $\therefore \frac{5x-1}{x^2(x-3)} = -\frac{14}{9x} + \frac{1}{3x^2} + \frac{14}{9(x-3)}$ $\Rightarrow \int \frac{5x-1}{x^2(x-3)}\,dx = \int\left(-\frac{14}{9x} + \frac{1}{3x^2} + \frac{14}{9(x-3)}\right)dx$ }..............[0.5] $= -\frac{14}{9}\log x + \frac{1}{3}\left(\frac{x^{-1}}{-1}\right) + \frac{14}{9}\log(x-3) + c$ $= \frac{14}{9}\log\frac{x-3}{x} - \frac{1}{3x} + c$ }[0.5]	

b) Find the direction cosines of the line perpendicular to lines AB and CD.  	[4]
Direction ratios of AB= −6, 2, 1 Direction ratios of CD=3, 2, −2 } ....[1] Let the direction ratios of the line perpendicular to AB and CD be a, b and c $-6a+2b+c=0$ and $3a+2b-2c=0$ } ....[1] $\frac{a}{-4-2}=\frac{b}{3-12}=\frac{c}{-12-6}=1$ } ....[1] $a=-6,\ b=-9,\ c=-18$ $or,\ a=2,\ b=3,\ c=6$ } ....[0.5] $l=\frac{2}{\sqrt{4+9+36}}=\frac{2}{7}$ $m=\frac{3}{\sqrt{4+9+36}}=\frac{3}{7}$ } ....[0.5] $n=\frac{6}{\sqrt{4+9+36}}=\frac{6}{7}$	
Question 13 a) Kuenga has a 4 m ladder. He leans the ladder against the wall at a distance of 1 m from the building. Find the angle subtended by the ladder with the ground in radians.	[2]
Let θ be the angle between the ladder and the ground. Then $\cos\theta=\frac{1}{4}$ } [1] $\Rightarrow \theta=\cos^{-1}\left(\frac{1}{4}\right)=75.52°$ } ....[0.5] $\theta=\frac{75.52}{180}\pi\ rad$ } ....[0.5]	

b)	Wangmo draws two cards, one after another, from the deck of a well shuffled pack of cards. What is the probability of drawing:	[5]
i)	either king or a queen in the first draw?	[1]

Ans:

Let A be drawing a king and B be drawing a queen

$P(A)=\frac{4}{52}, \; P(B)=\frac{4}{52}$ ....[0.5]

Since event A and B are mutually exclusive

$P(A\cup B)=P(A)+P(B)=\frac{4}{52}+\frac{4}{52}=\frac{2}{13}$ ....[0.5]

ii)	either a 10 or a club in the first draw?	[1]

Let A be drawing 10 and B be drawing of club.

P(A)=$\frac{4}{52}, P(B)=\frac{13}{52}, \; P(A\cap B)=\frac{1}{52}$ ....[0.5]

Since events A and B are not mutually exclusive

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$=\frac{4}{52}+\frac{13}{52}-\frac{1}{52}=\frac{16}{52}=\frac{4}{13}$ ....[0.5]

iii)	king or Queen of black in the first draw and Queen or Jack of red in the second draw if the first card drawn is replaced?	[1.5]

Let A be the events of drawing king or queen of black

P(A) = P(K or Q of black)=$\frac{2}{52}+\frac{2}{52}=\frac{4}{52}=\frac{1}{13}$ ....[0.5]

Let B be the events of drawing queen or jack of red

P(B) = P(Q or J of red) = $\frac{2}{52}+\frac{2}{52}=\frac{4}{52}=\frac{1}{13}$ ....[0.5]

$P(A\cap B)=$ P(K or Q of black)$\times$P(Q or J of red)

$=\frac{1}{13}\times\frac{1}{13}=\frac{1}{169}$ ....[0.5]

iv)	at least one diamond if the first card drawn is not replaced?	[1.5]

The possiblities are (Rand R) or $(\text{R and } \overline{\text{R}})$ or $(\overline{\text{R}} \text{ and R})$.............[0.5]

$=\frac{13}{52}\times\frac{12}{51}+\frac{13}{52}\times\frac{39}{51}+\frac{39}{52}\times\frac{13}{51}$ ....[0.5]

$=\frac{1170}{2652}=\frac{15}{34}$ ....[0.5]