MODEL AN BHUTAN HIGHER SECINDARY EDUCATION CERTIFICATE
DECEMBER 2023
SWER & MARKING SCHEME – BHSEC MATHEMATICS

GENERAL INSTRUCTIONS

1) The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the marking scheme are suggested answers. The content is thus indicative. If a student has given any other answer, which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weighting.
2) Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and strictly followed once the answers are thoroughly standardized.
3) If a question has parts, please award marks in the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin at the end of question. If a question does not have any parts, marks must be awarded in the left-hand margin.
4) If a candidate has attempted an extra question, it will be cancelled. The extra question attempted will be the last question in the sequence of the question paper.
5) Some examinees may attempt the questions giving equally correct answers in a different way. If the examiners are convinced that the response given by an examinee is genuinely correct, full weighting should be given.
6) If there are questions on distinction between two concepts, in such questions, sometimes some students give one aspect of the difference correctly and the other is either wrong or not given at all, no marks should be awarded.
7) There may be some questions requiring the examinees to give new ideas of their own or pass their own judgments and give valid justifications. In such cases, marks should be awarded for their efforts though there may be several possible answers.
8) If the questions ask for two features/characteristics/points but the examinee writes more than two features/characteristics/points, say, five of which the first is correct, second is incorrect, the best two should be assessed and the remaining should be ignored.
9) It is expected that the Marking Scheme be followed objectively for reliable marking. For instance, if an examinee scores 30 (BCSE) / 35 (BHSEC) marks, his/her marks should not be inflated to 35 (BCSE) / 40 (BHSEC) simply to pass him/her. Similarly, whenever an examinee has answered the question effectively, his/her marks should not be deducted unnecessarily. Do not hesitate to award full marks if the answer deserves it.
10) Marks should be awarded keeping in view the total marks of that particular question and not the total marks of the question paper. For example, if 1 mark is given to a question carrying 3 marks, even if nothing is correct, then that 1 mark constitutes 33% of the total marks ear-marked for this answer. This must be avoided. If a candidate fails in the subject by 1 to 3 marks, the marking may be reviewed

SECTION A [30 MARKS]
ANSWER ALL QUESTIONS

Question 1 **[30]**

Direction: For each question, there are four alternatives: A, B, C and D. Choose the correct alternative and circle it. Do not circle more than ONE alternative. If there are more than ONE choice circled, NO score will be awarded.

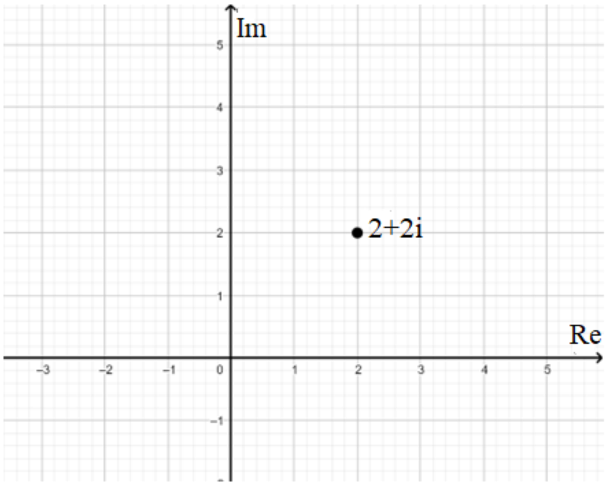
i) The polar form of complex number for the point shown in the graph is

A $-2\sqrt{2}cis\frac{\pi}{4}$.

B $-2\sqrt{2}cis\frac{\pi}{3}$.

(C) $2\sqrt{2}cis\frac{\pi}{4}$.

D $2\sqrt{2}cis\frac{\pi}{3}$.



Solution:

$2+2i$

$r=\sqrt{2^2+2^2}=\sqrt{8}=2\sqrt{2}$

C $\theta=\tan^{-1}\left|\frac{2}{2}\right|=\tan^{-1}(1)=\frac{\pi}{4}$

$\therefore rcis\theta=2\sqrt{2}cis\frac{\pi}{4}$

ii) Equation of a plane passes through the points $(-2,0,0),(0,1,0)$ *and* $(0,0,-1)$ is

A $x+2y+2z-2=0$

B $x-2y-2z-2=0$

(C) $x-2y+2z+2=0$

D $x+2y-2z-2=0$

Solution:

$\frac{x}{-2}+\frac{y}{1}+\frac{x}{-1}=1$

$\frac{-x+2y-2z}{2}=1$

C $-x+2y-2z=2$

$x-2y+2z=-2$

$Or, x-2y+2z+2=0$

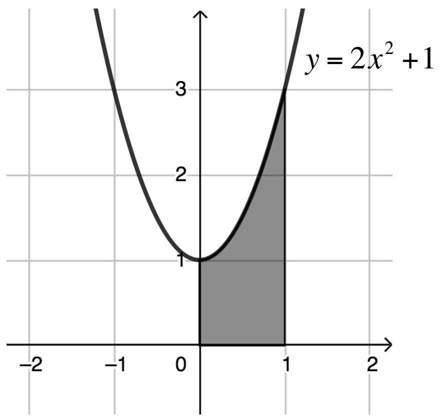
iii) What is the area of the shaded region?

A $\frac{1}{3}$ sq. units

B $\frac{2}{3}$ sq. units

C $\frac{4}{3}$ sq. units

(D) $\frac{5}{3}$ sq. units



Solution:

$$y = 2x^2 + 1$$

D $$\text{Area(A)} = \int_0^1 y\,dx = \int_0^1 \left(2x^2 + 1\right)dx$$

$$= \left[\frac{2}{3}x^3 + x\right]_0^1 = \frac{2}{3}(1)^3 + 1 - 0 = \frac{5}{3} \text{ sq. units}$$

iv) What is the degree and order of the differential equation $\frac{dy}{dx} - \frac{2}{dy/dx} = y$?

A Order: 1, degree: 1

(B) Order: 1, degree: 2

C Order: 2, degree: 1

D Order: 2, degree: 2

Solution:

$$\frac{dy}{dx} - \frac{2}{dy/dx} = y$$

$$\frac{\left(dy/dx\right)^2 - 2}{dy/dx} = y$$

B Order:1, degree:2

$$\left(\frac{dy}{dx}\right)^2 - 2 = y\frac{dy}{dx}$$

v) The principal value of $\tan^{-1}\left(\cot\frac{5\pi}{4}\right)$ is

(A) $\frac{\pi}{4}$.

B $\frac{5\pi}{4}$.

C $\frac{3\pi}{4}$.

D $\frac{\pi}{2}$.

Solution: $\tan^{-1}\left(\cot\frac{5\pi}{4}\right)=\tan^{-1}\left[\cot\left(\pi+\frac{\pi}{4}\right)\right]$ **A** $=\tan^{-1}\left(\cot\frac{\pi}{4}\right)$ $=\tan^{-1}\left[\tan\left(\frac{\pi}{2}-\frac{\pi}{4}\right)\right]=\frac{\pi}{2}-\frac{\pi}{4}=\frac{\pi}{4}$	
vi) If $A=\begin{pmatrix}3 & -1\\ -1 & 0\end{pmatrix}$ and $f(x)=x^2-3x+1$. Find $f(A)$. A $\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ (B) $\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}$ C $\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$ D $\begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}$ **Solution:** $A=\begin{pmatrix}3 & -1\\ -1 & 0\end{pmatrix}$ and $f(x)=x^2-3x+1$ **B** $f(A)=A^2-3A+I$ $=\begin{pmatrix}3 & -1\\ -1 & 0\end{pmatrix}\begin{pmatrix}3 & -1\\ -1 & 0\end{pmatrix}-3\begin{pmatrix}3 & -1\\ -1 & 0\end{pmatrix}+\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ $=\begin{pmatrix}10 & -3\\ -3 & 1\end{pmatrix}-\begin{pmatrix}9 & -3\\ -3 & 0\end{pmatrix}+\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ $=\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}+\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}=\begin{pmatrix}2 & 0\\ 0 & 2\end{pmatrix}$	
vii) There are 10 lamps in a hall. Each lamp can be switched on independently. The number of ways in which the hall can be illuminated is A 1024. (B) 1023. C 100. D 10!.	

Solution: **B** Each lamp can be switched on or off in 2 ways. A hall can be illuminated if at least one lamp is switched on. Therefore, the hall can be illuminated in $2^{10}-1=1023$ ways.	
viii) Find the value of P if the equation for the pair of straight lines is $x^2+2Pxy-6y^2+7x+31y-18=0$ and angle between the lines is 45^0. (A) $\frac{1}{2}$ B $\frac{3}{2}$ C $\frac{1}{4}$ D $\frac{3}{4}$ **Solution:** **A** Let $\cot^{-1} x=\theta$ $\tan^{-1} 45=\left\lvert 2\frac{\sqrt{p^2+6}}{-5}\right\rvert=2\frac{\sqrt{p^2+6}}{5}$ $1=2\frac{\sqrt{p^2+6}}{5}\Rightarrow\frac{5}{2}=\sqrt{p^2+6}$ $\frac{25}{4}-6=p^2\Rightarrow p=\frac{1}{2}$	
ix) $\int_0^{\frac{\pi}{2}}\cos\lvert x\rvert\,dx$ is A 0. (B) 1. C 2. D 3. **Solution:** **B** $\int_0^{\frac{\pi}{2}}\cos\lvert x\rvert\,dx=\int_0^{\frac{\pi}{2}}\cos x\,dx=\left[\sin x\right]_0^{\pi/2}=\sin\frac{\pi}{2}-\sin 0=1-0=1$ $\lvert x\rvert=\begin{cases}x, \text{ if } x\ge 0\\ -x, \text{ if } x<0\end{cases}$	

x) The gradient of a function, $\log(\sin x)$ at $x=\frac{\pi}{3}$ is A $-\frac{1}{\sqrt{3}}$. B $-\sqrt{3}$. C $\frac{1}{\sqrt{3}}$. (D) $\sqrt{3}$. **Solution:** $let\ \ y=\log(\sin x)$ $\frac{dy}{dx}=\frac{1}{\sin x}.\cos x=\cot x$ **D** $at\ \ x=\frac{\pi}{3}$ $\frac{dy}{dx}=\cot\frac{\pi}{3}=\sqrt{3}$	
xi) Distance travelled by an ant moving in a straight line at time t is described by $s=8t^3-6t^4$. What is the maximum acceleration with which the ant is moving? A 24 B 20 C 12 (D) 8 **Solution:** $s=8t^3-6t^4$ **D** $\text{velocity}(v)=\frac{ds}{dt}=24t^2-24t^3$ $\text{acceleration}(a)=\frac{dv}{dt}=48t-72t^2$ $\frac{da}{dt}=48-144t$ $\Rightarrow 48-144t=0$ $t=\frac{48}{144}=\frac{1}{3}$ $\frac{d^2a}{dt^2}=-144<0$, max So, $a=48\left(\frac{1}{3}\right)-72\left(\frac{1}{3}\right)^2=16-8=8$	

xii) Which of the points are collinear?

A $A(0, 2, 4)$, $B(4, 3, 1)$ and $C(2, 1, 2)$

(B) $A(1, 2, 3)$, $B(3, 8, 1)$ and $C(7, 20, -3)$

C $A(1, 4, 2)$, $B(-2, 1, 2)$ and $C(2, -3, 4)$

D $A(4, 2, 4)$, $B(10, 2, -2)$ and $C(2, 0, -4)$

Solution:

B

$A(1, 2, 3)$, $B(3, 8, 1)$ and $C(7, 20, -3)$

$$AB = \sqrt{(3-1)^2 + (8-2)^2 + (1-3)^2} = \sqrt{44} = 2\sqrt{11}$$

$$AC = \sqrt{(7-1)^2 + (20-2)^2 + (-3-3)^2} = \sqrt{396} = 6\sqrt{11}$$

$$BC = \sqrt{(7-3)^2 + (20-8)^2 + (-3-1)^2} = \sqrt{176} = 4\sqrt{11}$$

$$\Rightarrow AC = AB + BC$$

$\therefore A, B,$ and C are collinear points

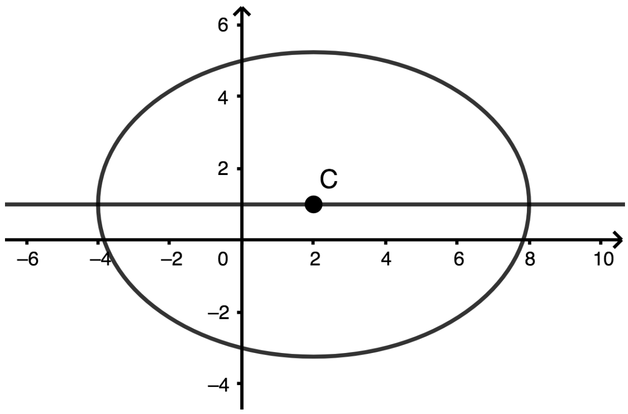
xiii) Find the equation for the given conic section whose eccentricity is $\frac{1}{\sqrt{2}}$.

A $\frac{(x-1)^2}{36} + \frac{(y-2)^2}{18} = 1$

B $\frac{(x-1)^2}{18} + \frac{(y-2)^2}{36} = 1$

(C) $\frac{(x-2)^2}{36} + \frac{(y-1)^2}{18} = 1$

D $\frac{(x-2)^2}{18} + \frac{(y-1)^2}{36} = 1$



Solution:

C

Centre : $(2, 1)$

a = distance from centre to a vertex = 6

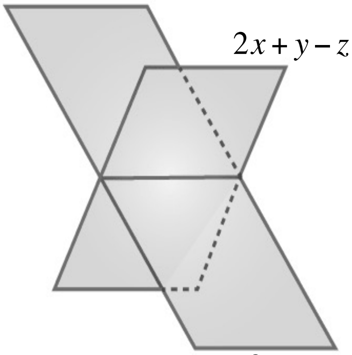
$$e = \frac{1}{\sqrt{2}}$$

$$b^2 = a^2\left(1 - e^2\right) = 36\left(1 - \frac{1}{2}\right) = 18$$

$$\frac{(x-2)^2}{36} + \frac{(y-1)^2}{18} = 1$$

xiv) The correlation between x and y is $\frac{2}{5}$ and the sum of the squares of the differences in ranks is 12. How many pairs were taken to calculate the correlation? (A) 5 B 6 C 8 D 10 **Solution: A** $r=\frac{2}{5}$ $\sum D^2=12$ $r=1-\frac{6\sum D^2}{n^3-n}$ $\frac{2}{5}=1-\frac{6(12)}{n^3-n}$ $\frac{6(12)}{n^3-n}=1-\frac{2}{5}=\frac{3}{5}$ $360=3n^3-3n$ $3n^3-3n-360=0$ When $n=5$ $3(5)^3-3(5)-360=0$ $0=0$ $\therefore n=5$	
xv) There are two bags. First bag contains 3 red, 4 yellow and 2 blue balls and the other bag contains 3 red, 3 yellow and 3 blue balls. A die is rolled. If a multiple of 3 is rolled, a ball is drawn from the first bag, otherwise a ball is drawn from the second bag. What is the probability of drawing a yellow ball? A $\frac{20}{81}$ B $\frac{19}{54}$ (C) $\frac{10}{27}$ D $\frac{7}{18}$	

Solution: Let multiple of 3 = *3, and Yellow = Y **C** $P(*3\cap Y)=\frac{2}{6}\times\frac{4}{9}=\frac{8}{54}$ Or $P\left(\overline{*3}\cap Y\right)=\frac{4}{6}\times\frac{3}{9}=\frac{12}{54}$ $\Rightarrow P(Y)=\frac{8}{54}+\frac{12}{54}=\frac{20}{54}=\frac{10}{27}$	
SECTION B [70 MARKS] **ATTEMPT ANY TEN QUESTIONS**	
Question 2	
a) If $\sqrt{\frac{y}{x}}+\sqrt{\frac{x}{y}}=6$, show that $\frac{dy}{dx}=\frac{x-17y}{17x-y}$.	**[4]**
Solution: $\sqrt{\frac{y}{x}}+\sqrt{\frac{x}{y}}=6$ $\left.\begin{array}{l}\Rightarrow \frac{y+x}{\sqrt{xy}}=6\\ x+y=6\sqrt{xy}\end{array}\right\}0.5$ $\left.\begin{array}{l}\text{Sq. both sides}\\ x^2+2xy+y^2=36xy\\ x^2-34xy+y^2=0\end{array}\right\}0.5$ $\left.\begin{array}{l}\text{Diff. w.r.t. x}\\ 2x-34\left(x\frac{dy}{dx}+y\right)+2y\frac{dy}{dx}=0\end{array}\right\}1$ $\left.\frac{dy}{dx}\left(-34x+2y\right)=-2x+34y\right\}0.5$ $\left.\frac{dy}{dx}=\frac{-2x+34y}{-34x+2y}=\frac{-x+17y}{-17x+y}=\frac{x-17y}{17x-y}\right\}0.5$	

b) Prove the theorem $adj(AB) = (adj\,B)(adj\,A)$ with an example.		**[3]**
Solution: $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ & $B = \begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix}$ $AB = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} 0 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 3 & 11 \end{bmatrix}$ } 1 $adj(AB) = \begin{bmatrix} 11 & -4 \\ -3 & 2 \end{bmatrix}$ } 0.5 $(adj\,B)(adj\,A) = \begin{bmatrix} 1 & -2 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 3 & -2 \\ -4 & 1 \end{bmatrix} = \begin{bmatrix} 11 & -4 \\ -3 & 2 \end{bmatrix}$ } 1.5		
Question 3		
a) Find the equation of the plane through the intersection of given planes and passing through the point (2, 1, -1).  		**[4]**
Solution: Equation of a plane passing through the intersection of $2x+y-z-1=0$ and $3x-y-z+2=0$ is given by $2x+y-z-1+k(3x-y-z+2)=0$ $(2+3k)x+(1-k)y+(-1-k)z+(-1+2k)=0 --- eq(1)$ } 0.5 (1) passes through $(2,1,-1)$, so (1) becomes $(2+3k)2+(1-k)1+(-1-k)(-1)+(-1+2k)=0$ } 1 $4+6k+1-k+1+k-1+2k=0$ $8k+5=0$ $k=-\frac{5}{8}$ } 1		

Substituting (1), we get

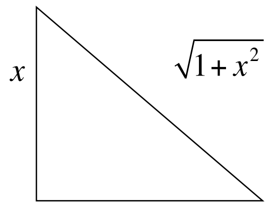
$$\left[2+3\left(-\frac{5}{8}\right)\right]x+\left(1+\frac{5}{8}\right)y+\left(-1+\frac{5}{8}\right)z+\left[-1+2\left(-\frac{5}{8}\right)\right]=0 \quad \Big\} 1$$

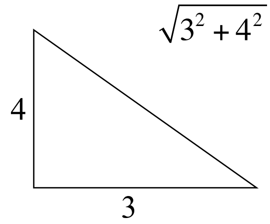
$$\left.\begin{aligned}&\frac{1}{8}x+\frac{13}{8}y-\frac{3}{8}z-\frac{18}{8}=0\\&x+13y-3z-18=0\end{aligned}\right\}0.5$$

b) Solve: $\cos\left(\sin^{-1}\frac{x}{\sqrt{1+x^2}}\right)=\sin\left(\cot^{-1}\frac{3}{4}\right)$ **[3]**

Solution:

$$\cos\left(\sin^{-1}\frac{x}{\sqrt{1+x^2}}\right)=\sin\left(\cot^{-1}\frac{3}{4}\right)$$





$$\sqrt{3^2+4^2}=5\Big\}0.5$$

$$\sqrt{\sqrt{1+x^2}^{\,2}-x^2}=\sqrt{1+x^2-x^2}=1\Big\}0.5$$

$$\cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right)=\sin\left(\sin^{-1}\frac{4}{5}\right)\Big\}0.5\times 2=1$$

$$\left.\begin{aligned}&\frac{1}{\sqrt{1+x^2}}=\frac{4}{5}\\&5=4\sqrt{1+x^2}\\&\text{sq. both sides}\\&25=16\left(1+x^2\right)\end{aligned}\right\}0.5$$

$$\left.\begin{aligned}&25=16+16x^2\\&\Rightarrow 16x^2=9\\&x^2=\frac{9}{16}\\&x=\pm\frac{3}{4}\end{aligned}\right\}1$$

Question 4

a) Check and prove with an example whether the given statement $\int\frac{f'(x)}{f(x)}dx=\log\left|f(x)\right|+c$ is true or false. **[3]**

Solution:

True $\}1$

$$\int \frac{2x}{1+x^2}dx = \log\left(1+x^2\right)+c$$

Where $f(x)=1+x^2$ & $f'(x)=2x$ $\Big\}2$

b) Calculate the line of best fit for the following data and then estimate the value of height when the weight is 56 kg. **[4]**

Height(cm)	156	154	150	172	150	168	170	168
Weight(kg)	60	64	61	67	55	52	70	67

Solution:

Height(cm)	**156**	**154**	**150**	**172**	**150**	**168**	**170**	**168**
Weight(kg)	**60**	**64**	**61**	**67**	**55**	**52**	**70**	**67**
$x-\bar{x}$	**-5**	**-7**	**-11**	**11**	**-11**	**7**	**9**	**7**
$y-\bar{y}$	**-2**	**2**	**-1**	**5**	**-7**	**-10**	**8**	**5**
$\left(x-\bar{x}\right)\left(y-\bar{y}\right)$	**10**	**-14**	**11**	**55**	**77**	**-70**	**72**	**35**
$\left(y-\bar{y}\right)^2$	**4**	**4**	**1**	**25**	**49**	**100**	**64**	**25**

$\Big\}2$

$$\sum\left(x-\bar{x}\right)\left(y-\bar{y}\right)=176$$

$$\sum\left(y-\bar{y}\right)^2=272$$

$$\left.\begin{aligned}\bar{x}&=\frac{\sum x}{n}=\frac{1288}{8}=161\\ \bar{y}&=\frac{\sum y}{n}=\frac{496}{8}=62\end{aligned}\right\}0.5$$

$$\left.b_{xy}=\frac{\sum\left(x-\bar{x}\right)\left(y-\bar{y}\right)}{\sum\left(y-\bar{y}\right)^2}=\frac{176}{272}=\frac{11}{17}\right\}0.5$$

$$\left.\begin{aligned}&X\ on\ Y: x-\bar{x}=b_{xy}\left(y-\bar{y}\right)\\ &\quad x-161=\frac{11}{17}\left(y-62\right)\\ &\quad 17x-2737=11y-682\\ &17x=11y+2055\end{aligned}\right\}0.5$$

$$\left.\begin{aligned}&\text{When y = 56kg}\\ &17x=11(56)+2055\\ &x\approx 157\\ &\therefore Height=157\text{ cm}\end{aligned}\right\}0.5$$

Question 5

a) Find the value of P and Q, if the following equation represents a pair of perpendicular lines $6x^2+5xy-Py^2+7x+Qy-5=0$. **[4]**

Solution:

If two lines are perpendicular, then $a+b=0$

$6x^2+5xy-Py^2+7x+Qy-5=0$

$ax^2+2hxy+by^2+2gx+2fy+c=0$

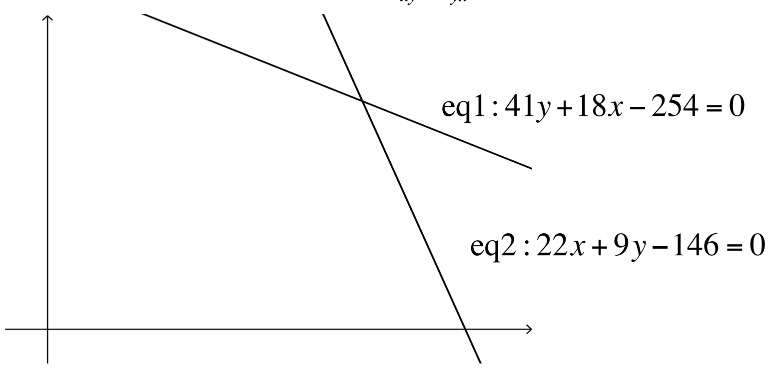
$$\left.\begin{array}{l} a=6,\ b=-P \\ 6-P=- \\ P=6 \end{array}\right\}1$$

$abc+2fgh-af^2-bg^2-ch^2=0$

$$\left.a=6,\ b=-6,\ h=\frac{5}{2},\ g=\frac{7}{2},\ f=\frac{Q}{2},\ c=-5\right\}1$$

$$\left.\begin{array}{l} 180+35\left(\frac{Q}{4}\right)-6\left(\frac{Q^2}{4}\right)+6\left(\frac{49}{4}\right)+5\left(\frac{25}{4}\right)=0 \\ \frac{720+35Q-6Q^2+294+125}{4}=0 \\ 6Q^2-35Q-1139=0 \\ (6Q+67)(Q-17)=0 \\ Q=\frac{-67}{6},17 \end{array}\right\}2$$

b) The given equations eq1 and eq2 are regression equations of X on Y and Y on X respectively. Is the statement true? Compute $b_{xy}, b_{yx}, \overline{x}$ and $\overline{y}$. **[3]**



Solution: $\text{eq1}: 41y+18x-254=0\ (X\ on\ Y)$ $b_{xy}=-\frac{41}{18}$ $\text{eq2}: 22x+9y-146=0\ (Y\ on\ X)$ $b_{yx}=-\frac{22}{9}$ $r=\pm\sqrt{b_{xy}\times b_{yx}}=-\sqrt{-\frac{41}{18}\times-\frac{22}{9}}=-2.36\ but\ -1\le r\le 1$ }1 So, eq1 is Y on X and eq2 is X on Y, hence the statement is not true $\therefore b_{yx}=-\frac{18}{41}\ and\ b_{xy}=-\frac{9}{22}$ $r=\pm\sqrt{b_{yx}\times b_{xy}}=-\sqrt{-\frac{18}{41}\times-\frac{9}{22}}=-0.42$ }1 $18x+41y-254=0$ $22x+9y-146=0$ $\frac{x}{3700}=\frac{y}{2960}=\frac{1}{740}$ $x=\bar{x}=\frac{3700}{740}=5$ $y=\bar{y}=\frac{3960}{740}=4$ }1	
Question 6	
a) A school literary committee has organized 3 literary activities to observe the literary week. For each activity there is only one prize. If the same 10 students have participated in all three activities, how can the prize be distributed if, i) a student can receive any number of prizes? ii) a student cannot receive all the three prizes?	**[3]**
Solution: i) 1st literary-10 2nd literary-10 3rd literary-10 $\text{Total}=10\times10\times10=1000\}1.5$ ii) Cannot receive all prizes = receive any number of prizes – receive all three prizes =1000 –10 = 990 }1.5	

b) Find the locus of a point such that the difference of its distances from $(6,-2)$ and $(-4,-2)$ is always equal to 8 and then identify the locus of the point.	**[4]**
Solution: Let P(x, y) be the moving point Let A be $(6,-2)$ and B be $(-4,-2)$. Then $\left.PA-PB=8\right\\}0.5$ $\left.\begin{array}{l}\sqrt{(x-6)^2+(y+2)^2}-\sqrt{(x+4)^2+(y+2)^2}=8\\\\ \sqrt{(x-6)^2+(y+2)^2}=8+\sqrt{(x+4)^2+(y+2)^2}\end{array}\right\\}0.5$ $\left.\begin{array}{l}\text{Sq. both sides and expanding}\\\\ x^2-12x+36+y^2+4y+4=64+16\sqrt{(x+4)^2+(y+2)^2}+x^2+8x+16+y^2+4y+4\\\\ -12x+36=64+16\sqrt{(x+4)^2+(y+2)^2}+8x+16\end{array}\right\\}1$ $\left.\begin{array}{l}-20x-44=16\sqrt{(x+4)^2+(y+2)^2}\\\\ -5x-11=4\sqrt{(x+4)^2+(y+2)^2}\end{array}\right\\}0.5$ Sq. both sides and expanding $\left.25x^2+110x+121=16(x^2+8x+16+y^2+4y+4)\right\\}0.5$ $\left.\begin{array}{l}25x^2+110x+121=16x^2+128x+16y^2+64y+320\\\\ 9x^2-18x-16y^2-64y-199=0\end{array}\right\\}0.5$ $\left.\begin{array}{l}a=9, b=-16, h=0\\\\ h^2-ab=0-(9)(-16)=144>0, \text{hence hyperbola.}\end{array}\right\\}0.5$	
Question 7	
a) Solve: $\cos^{-1}\left(\frac{dy}{dx}\right)=x+y$	**[3]**
Solution: $\cos^{-1}\left(\frac{dy}{dx}\right)=x+y$ $\left.\frac{dy}{dx}=\cos(x+y)\right\\}0.5$ $\left.\begin{array}{l}Let\ x+y=z\\\\ diff.\ w.r.t.\ x\\\\ 1+\frac{dy}{dx}=\frac{dz}{dx}\\\\ \frac{dy}{dx}=\frac{dz}{dx}-1\end{array}\right\\}0.5$	

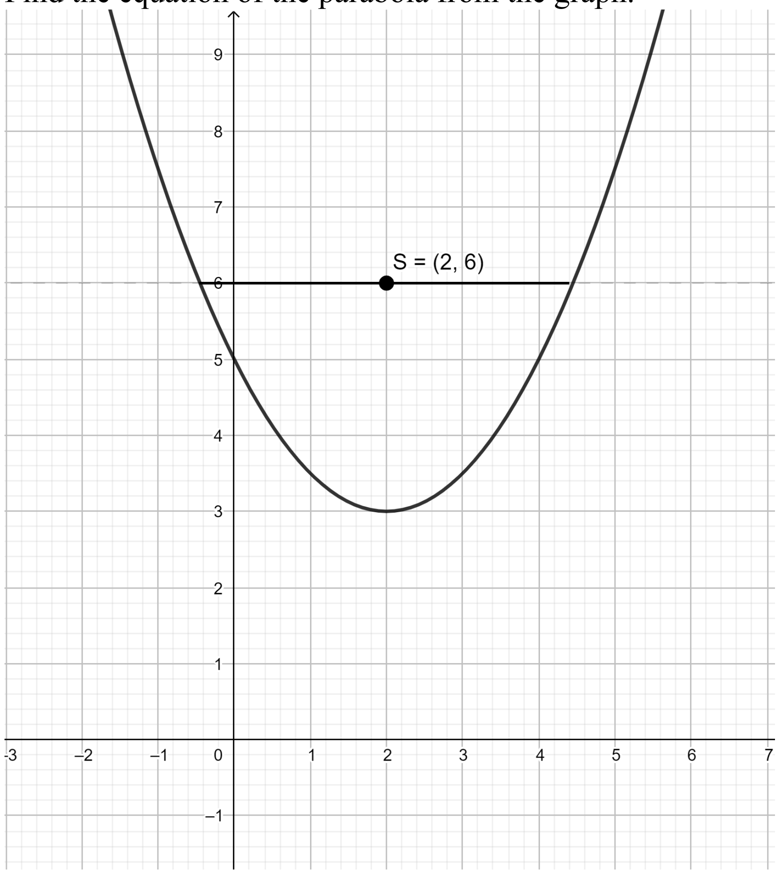
$$\left.\begin{aligned}&\Rightarrow \frac{dz}{dx}-1=\cos z\\&\frac{dz}{dx}=\cos z+1\\&\frac{1}{1+\cos z}dz=dx\end{aligned}\right\}0.5$$ $$\left.\begin{aligned}&\Rightarrow \int\frac{1}{1+\cos z}dz=\int dx\\&\int\frac{1}{2\cos^2\frac{z}{2}}dz=x+c\end{aligned}\right\}0..5$$ $$\left.\frac{1}{2}\int\sec^2\frac{z}{2}dz=x+c\right\}0.5$$ $$\left.\begin{aligned}&\frac{1}{2}\times\frac{\tan\frac{z}{2}}{\left(\frac{1}{2}\right)}=x+c\\&\tan\frac{z}{2}=x+c\end{aligned}\right\}0.5$$	
b) The ages of 7 married couples were collected to find the correlation between the ages of husbands (x) and wives (y) and the following results were listed: $\sum x=263, \sum y=221, \sum xy=8929, \sum x^2=10591\sum y^2=7647$. On further inspection, the pair $(x=35, y=23)$ was recorded incorrectly. The correct pair was $(x=38, y=33)$. What might be the correct correlation between the ages of husbands and wives? Interpret the result. Provide the possible list of ages of 7 wives that sum up to 231.	**[4]**
Solution: $$\left.\begin{aligned}&\sum x=263-35+38=266\\&\sum y=221-23+33=231\end{aligned}\right\}0.5$$ $$\left.\begin{aligned}&\sum xy=8929-(35\times 23)+(38\times 33)=9378\\&\sum x^2=10591-35^2+38^2=10810\\&\sum y^2=7647-23^2+33^2=8207\end{aligned}\right\}1$$ $$\left.\begin{aligned}r&=\frac{n\sum xy-\sum x\sum y}{\sqrt{n\sum x^2-\left(\sum x\right)^2}\sqrt{n\sum y^2-\left(\sum y\right)^2}}\\&=\frac{7(9378)-(266)(231)}{\sqrt{7(10810)-(266)^2}\sqrt{7(8207)-(231)^2}}=\frac{4200}{\sqrt{4914}\sqrt{4088}}=0.937\end{aligned}\right\}1$$ There is a very high positive correlation between the ages of husband and wife---0.5 Possible list of ages of 7 wives that sum up to 231 are: 28, 30, 32, 31, 35, 36, 39---1	

Question 8	
a) Evaluate: $\int_{-\pi}^{\pi} \sin^3 x\, dx$	**[2]**
Solution: $f(x) = \sin^3 x$ $\left.\begin{array}{l} f(-x) = \sin^3(-x) = -\sin^3 x \\ \Rightarrow f(x) \text{ is a odd function} \end{array}\right\}1$ $\therefore \int_{-\pi}^{\pi} \sin^3 x\, dx = 0 \Big\} 1$	
b) Match the following and write your answer in the table given below.	**[2]**

Function	Range
1. $\sin^{-1} x$	a. $[0, \pi]$
2. $\cos^{-1} x$	b. $[0, \pi] - \left\{\frac{\pi}{2}\right\}$
3. $\sec^{-1} x$	c. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$
4. $co\sec^{-1} x$	d. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Solution:

Function	Range
1. $\sin^{-1} x$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$........}0.5
2. $\cos^{-1} x$	$[0, \pi]$........}0.5
3. $\sec^{-1} x$	$[0, \pi] - \left\{\frac{\pi}{2}\right\}$........}0.5
4. $co\sec^{-1} x$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$ ....}0.5

c) A mother, a father and a child line up at random for a family picture. What is the probability of the child being on one end if the father is in the middle?	**[3]**
Solution: A: child on one end B: father in the middle If mother (M), father (F) and child (C) line for family picture, then Sample space={MFC, MCF, FMC, FCM, CMF, CFM} A={MFC, FMC, CMF, CFM} B={MFC, CFM} A ∩ B={MFC, CFM} } 1 $P(A \cap B) = \frac{2}{6} = \frac{1}{3}$ $P(B) = \frac{2}{6} = \frac{1}{3}$ } 1 $\therefore P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{\frac{1}{3}}{\frac{1}{3}} = 1$ } 1	
Question 9	
a) Find the equation of the parabola from the graph. 	**[3]**

Solution: $x^2 = 4ay.....(i)$ From graph, vertex is at $(2,3)$ distance from vertex to focus $= a = 3$ } 1 $subs.\ eq.\ (i)$ $(x-2)^2 = 4\times 3(y-3)$ } 1 $x^2 - 4x + 4 = 12y - 36$ $x^2 - 4x - 12y + 40 = 0$ } 1	
b) Evaluate: $\int \frac{\sin x \cos x}{\cos^2 x(\sin x + 1)}dx$	**[4]**
Solution: $\int \frac{\sin x \cos x}{\cos^2 x(\sin x+1)}dx$ $\int \frac{\sin x \cos x}{(1-\sin^2 x)(\sin x+1)}dx$ } 0.5 $Let\ \sin x = t$ $Diff.\ w.r.t.\ x$ $\cos x dx = dt$ $\Rightarrow \int \frac{tdt}{(1-t^2)(t+1)} = \int \frac{tdt}{(1-t)(t+1)^2}$ } 0.5 Let $\frac{t}{(1-t)(t+1)^2} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{(1+t)^2}$ $\Rightarrow t = A(1+t)^2 + B(1-t)(1+t) + C(1-t)$ $t = A(1+t)^2 + B(1-t^2) + C(1-t)$ } 0.5 $When\ t=1$ $When\ t=-1$ $1 = 4A$ $-1 = 2C$ $A = \frac{1}{4}$ $C = -\frac{1}{2}$ $Equate\ Coefficient\ of\ t^2$ $0 = A - B$ $B = A = \frac{1}{4}$ } 1 $\Rightarrow \frac{t}{(1-t)(t+1)^2} = \frac{1}{4(1-t)} + \frac{1}{4(1+t)} - \frac{1}{2(1+t)^2}$ } 0.5	

$$\Rightarrow \int\left[\frac{1}{4(1-t)}+\frac{1}{4(1+t)}-\frac{1}{2(1+t)^2}\right]dt$$

$$=-\frac{1}{4}\log(1-t)+\frac{1}{4}\log(1+t)+\frac{1}{2}(1+t)^{-1}+c$$

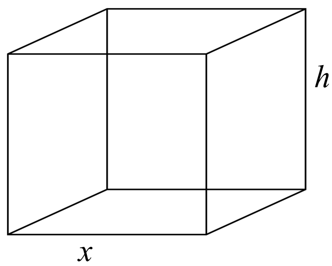
$$=-\frac{1}{4}\log(1-\sin x)+\frac{1}{4}\log(1+\sin x)+\frac{1}{2}(1+\sin x)^{-1}+c$$

Or, $$=\frac{1}{4}\log\left[\frac{1+\sin x}{1-\sin x}\right]+\frac{1}{2(1+\sin x)}+c$$

} 1

Question 10

a) A man decide to construct an open tank with a square base to fold P m³ of water for his new house. Show that the cost of the material will be least if $h=\frac{1}{2}x$. **[3]**



Solution:

$$\text{Volume}=x^2h$$
$$P=x^2h$$
$$\Rightarrow h=\frac{P}{x^2}$$
} 0.5

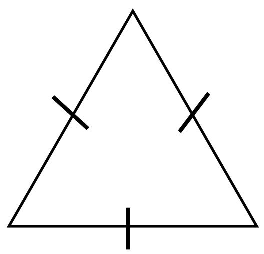
$$\text{Surface area}(A)=x^2+4xh$$
$$A=x^2+4x\frac{P}{x^2}=x^2+\frac{4P}{x}$$
} 0.5

$$\frac{dA}{dx}=2x-\frac{4P}{x^2}$$
} 0.5

$$2x-\frac{4P}{x^2}=0$$
$$2x^3-4P=0$$
$$2x^3=4P$$
$$x^3=2P$$
$$x=\sqrt[3]{2P}$$
} 0.5

$$\frac{d^2A}{dx^2}=2+\frac{8P}{x^3}$$
$$\left.\frac{d^2A}{dx^2}\right|_{x^3=2P}=2+\frac{8P}{2P}=6>0,\ \text{min}$$
} 0.5

$$h=\frac{P}{x^2}=\frac{Px}{x^3}=\frac{Px}{2P}=\frac{x}{2},\ \text{hence proved.}$$
} 0.5

b) Find the equation of the lines parallel to the lines $x^2+xy-6y^2+7x+31y-18=0$ and passing through the point $(-1,2)$.	[4]
Solution: Equation of a line parallel to lines $x^2+xy-6y^2+7x+31y-18=0$ is $x^2+xy-6y^2=0.---eq(1)$ $\}0.5$ *eq(1)* passes through the point $(-1,2)$ $(x+1)^2+(x+1)(y-2)-6(y-2)^2=0$ $\}0.5$ $x^2+2x+1+xy-2x+y-2-6(y^2-4y+4)=0$ $\}0.5$ $x^2+1+xy+y-2-6y^2+24y-24=0$ $x^2+xy-6y^2+25y-25=0$ $\}0.5$ From the lines $x^2+xy-6y^2+25y-25=0$ $a=1$ $b=y$ $c=-6y^2+25y-25$ $x=\frac{-y\pm\sqrt{y^2-4(-6y^2+25y-25)}}{2}$ $2x=-y\pm\sqrt{y^2+24y^2-100y+100}$ $2x=-y\pm\sqrt{(5y-10)^2}$ $2x=-y\pm(5y-10)$ $\}1$ $2x-4y+10=0,\quad 2x+6y-10=0$ $\}1$	
Question 11	
a) A religious body is planning to construct a unique religious structure as shown in the diagram. Find the value of P, if coordinates of the vertices are $(4,2,4),(10,P,-2)$ and $(2,0,-4)$ 	[3]

Solution:

Let $A=(4,2,4)$, $B=(10,P,-2)$, $C=(2,0,4)$

If it is equilateral triangle, all sides are equal

$AB = BC \quad \}1$

$$\left.\begin{aligned}&\sqrt{(10-4)^2+(P-2)^2+(-2-4)^2}=\sqrt{(2-4)^2+(0-2)^2+(-4-4)^2}\\&\sqrt{36+(P-2)^2+36}=\sqrt{4+4+64}\end{aligned}\right\}1$$

$$\left.\begin{aligned}&\textit{Squaring on both sides}\\&(P-2)^2+72=72\\&P-2=0\\&P=2\end{aligned}\right\}1$$

b) Find the number of (i) combinations (ii) permutations of four letters taken from the word CERTIFICATE? **[4]**

Solution:

CERTIFICATE

(CC), (EE), (TT), (I I) R E A

i) Combination ……………………}2

a) 2 alike, 2 alike = ${}^4C_2=6$

b) 2 alike, 2 different = ${}^4C_1\times{}^3C_2=12$

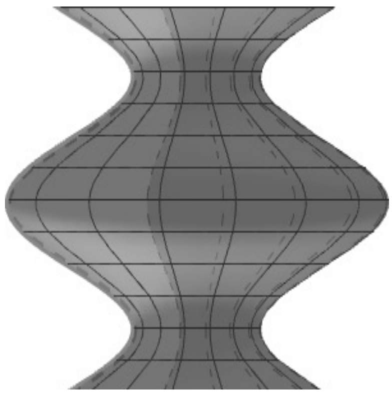
c) All 4 different = ${}^7C_4=35$

Total combination = $6+12+35=53$

ii) Permutation = $6\times\frac{4!}{2!2!}+12\times\frac{4!}{2!}+35\times4!=36+144+840=1020 \quad \}2$

Question 12

a) A company wants to create a vessel of a given shape which is formed by revolving the graph of $y=\sin 2x+2$ about the x - axis over the interval $0\le x\le\pi$. What is the volume of the vessel? **[3]**

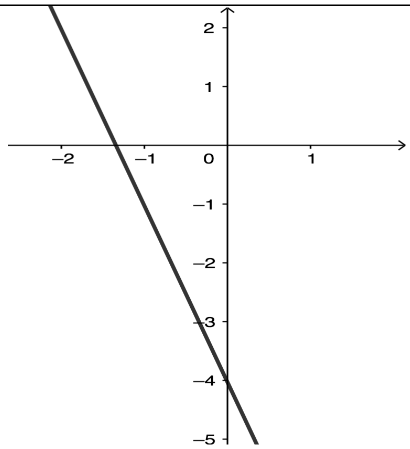


Solution: $\text{Volume} = \pi\int_0^\pi y^2 dx = \pi\int_0^\pi (\sin 2x+2)^2 dx$ $= \pi\int_0^\pi (\sin^2 2x + 4\sin 2x + 4)dx$ } 0.5 $= \pi\int_0^\pi \left(\frac{1-\cos 4x}{2} + 4\sin 2x + 4\right)dx$ } 0.5 $= \pi\left[\frac{1}{2}\left(x - \frac{\sin 4x}{4}\right) - 4\frac{\cos 2x}{2} + 4x\right]_0^\pi$ $= \pi\left[\frac{x}{2} - \frac{\sin 4x}{8} - 2\cos 2x + 4x\right]_0^\pi$ } 1 $= \pi\left[\frac{\pi}{2} - \frac{\sin 4\pi}{8} - 2\cos 2\pi + 4\pi - \left(\frac{0}{2} - \frac{\sin 4(0)}{8} - 2\cos 2(0) + 4(0)\right)\right]$ $= \pi\left[\frac{\pi}{2} - 0 - 2(1) + 4\pi - \left(0 - 0 - 2(1) + 0\right)\right]$ } 0.5 $= \pi\left[\frac{\pi}{2} - 2 + 4\pi + 2\right]$ $= \frac{9\pi^2}{2}$ cu. units } 0.5	
b) A boy throws a stone upward and the velocity after t seconds is $v(t) = at^2 - bt + c, 0 \le t \le 8,$ where *a, b* and *c* are constants. It was found that at time $t = 1$, 2, and 4 seconds, the velocity of the stone was 2, 3 and 11 m/s respectively. Find the speed at 6 seconds using matrix method.	**[4]**
Solution: $v(1) = 2$ $a - b + c = 2,$ $v(2) = 3$ $4a - 2b + c = 3,$ and $v(3) = 11$ $9a - 3b + c = 11$ } 0.5	

$\Rightarrow a-b+c=2$ $4a-2b+c=3$ $9a-3b+c=11$ } are the three simultaneous equations. $\begin{bmatrix}1 & -1 & 1\\ 4 & -2 & 1\\ 16 & -4 & 1\end{bmatrix}\begin{bmatrix}a\\ b\\ c\end{bmatrix}=\begin{bmatrix}2\\ 3\\ 11\end{bmatrix}$ $AX=B$ } 0.5 $\lvert A\rvert=1(-2+4)+1(4-16)+1(-16+32)=2-12+16=6$ } 0.5 $adjA=\begin{matrix}-2 & 1 & 4 & -2\\ -4 & 1 & 16 & -4\\ -1 & 1 & 1 & -1\\ -2 & 1 & 4 & -2\end{matrix}=\begin{bmatrix}2 & 12 & 16\\ -3 & -15 & -12\\ 1 & 3 & 2\end{bmatrix}'=\begin{bmatrix}2 & -3 & 1\\ 12 & -15 & 3\\ 16 & -12 & 2\end{bmatrix}$ } 1 $A^{-1}=\frac{1}{\lvert A\rvert}(adjA)=\frac{1}{6}\begin{bmatrix}2 & -3 & 1\\ 12 & -15 & 3\\ 16 & -12 & 2\end{bmatrix}$ } 0.5 $X=A^{-1}B=\frac{1}{6}\begin{bmatrix}2 & -3 & 1\\ 12 & -15 & 3\\ 16 & -12 & 2\end{bmatrix}\begin{bmatrix}2\\ 3\\ 11\end{bmatrix}=\frac{1}{6}\begin{bmatrix}6\\ 12\\ 18\end{bmatrix}=\begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$ } 0.5 $\therefore v(t)=t^2-2t+3$ So, $v(6)=6^2-2(6)+3=27\ m/s$ } 0.5	
Question 13	
a) If $z=x+yi, \omega=\frac{1+zi}{2z+3}$ and $\lvert\omega\rvert=1$, find the locus of z and illustrate it on a complex plane.	**[3]**
Solution: $\omega=\frac{1+zi}{z+3}=\frac{1+(x+yi)i}{x+yi+3}=\frac{1+xi-y}{(x+3)+yi}=\frac{(1-y)+xi}{(x+3)+yi}$ $\lvert\omega\rvert=1$ } 0.5 $\Rightarrow\left\lvert\frac{(1-y)+xi}{(x+3)+yi}\right\rvert=1$ $\frac{\lvert(1-y)+xi\rvert}{\lvert(x+3)+yi\rvert}=1$ $(1-y)^2+x^2=(x+3)^2+(y)^2$ } 0.5	

$$\left.\begin{aligned}&1-2y+y^2+x^2=x^2+6x+9+y^2\\&1-2y-6x-9=0\\&-2y-6x-8=0\\&\text{Or, } y+3x+4=0\end{aligned}\right\}0.5$$

$$\left.\begin{aligned}&y=-3x-4\\&Slope=-3\\&y-\text{int}=-4\end{aligned}\right\}0.5$$



So it represents points on the line whose slope is (– 3) and the coordinates of y-intercept is (0, – 4).

Graph. ------------- 1

b) Prove that $x\dfrac{d^2y}{dx^2}+(1-4y)\dfrac{dy}{dx}=0$ if $y=\tan\left(\log x^2\right)$. **[4]**

Solution:

$$y=\tan\left(\log x^2\right)$$

$$\left.\frac{dy}{dx}=\sec^2(\log x^2)\frac{1}{x^2}(2x)=\frac{2\sec^2(\log x^2)}{x}\right\}1$$

$$\left.\frac{d^2x}{dx^2}=\frac{x\left[4\sec(\log x^2)\sec(\log x^2)\tan(\log x^2)\frac{1}{x^2}(2x)\right]-2\sec^2(\log x^2)(1)}{x^2}\right\}1$$

$$\left.\begin{aligned}&=\frac{8\sec^2(\log x^2)\tan(\log x^2)-2\sec^2(\log x^2)}{x^2}\\&=\frac{2\sec^2(\log x^2)\left[4\tan(\log x^2)-1\right]}{x^2}\end{aligned}\right\}0.5$$

$$\left.\begin{aligned}LHS=x\frac{d^2y}{dx^2}+(1-4y)\frac{dy}{dx}&=x\left(\frac{2\sec^2(\log x^2)\left[4\tan(\log x^2)-1\right]}{x^2}\right)\\&\quad+\left(1-4\tan(\log x^2)\right)\left(\frac{2\sec^2(\log x^2)}{x}\right)\end{aligned}\right\}0.5$$

$$\left.\begin{aligned}&=\frac{2\sec^2(\log x^2)\left[4\tan(\log x^2)-1\right]}{x}-\frac{2\sec^2(\log x^2)\left[4\tan(\log x^2-1\right]}{x}\\&=0=RHS\text{, hence proved.}\end{aligned}\right\}1$$

OR, $\left.\begin{array}{l} y=\tan\left(\log x^2\right) \\ \dfrac{dy}{dx}=\sec^2(\log x^2)\dfrac{1}{x^2}(2x)=\dfrac{2\sec^2(\log x^2)}{x} \end{array}\right\}1$ $\left.\begin{array}{l} x\dfrac{dy}{dx}=2\sec^2(\log x^2) \\ \Rightarrow x\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}(1)=2\left[2\sec(\log x^2)\sec(\log x^2)\tan(\log x^2)\dfrac{1}{x^2}(2x)\right] \end{array}\right\}1$ $\left.\begin{array}{l} x\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}=\dfrac{8\sec(\log x^2)\sec(\log x^2)\tan(\log x^2)}{x} \\ \qquad =\dfrac{8\sec^2(\log x^2)\tan(\log x^2)}{x} \end{array}\right\}0.5$ $\qquad =4\left[\dfrac{2\sec^2(\log x^2)}{x}\right]\times\tan(\log x^2)\Big\}0.5$ $x\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}=4\dfrac{dy}{dx}\times y=4y\dfrac{dy}{dx}\Big\}0.5$ $\left.\begin{array}{l} x\dfrac{d^2y}{dx^2}+\dfrac{dy}{dx}-4y\dfrac{dy}{dx}=0 \\ x\dfrac{d^2y}{dx^2}+(1-4y)\dfrac{dy}{dx}=0,\text{ hence proved.} \end{array}\right\}0.5$	