SECTION A [30 MARKS]

ANSWER **ALL** QUESTION

Question 1 **[30]**

Direction: For each question, there are FOUR responses: A, B, C, and D. Choose the corresponding letter of your response and CIRCLE it neatly. NO score will be awarded if you circle more than ONE letter.

i. For every matrix $A_{m\times n}$, there exist a matrix B such that $A_{m\times n} + B = 0$. Then matrix B is

A $A_{n\times m}$.

B $A_{m\times n}$.

C $-A_{n\times m}$.

D $-A_{m\times n}$.

ii. What is $\frac{dy}{dx}$, if $x = \cos t$ and $y = e^{\log_e \sin t}$?

A $\cot t$

B $\tan t$

C $-\cot t$

D $-\tan t$

iii. Find the principal value of $\sin^{-1}\left(\cos\frac{2\pi}{3}\right) + \cos^{-1}\left(\sin\frac{3\pi}{4}\right)$.

A $\frac{\pi}{4}$

B $\frac{5\pi}{12}$

C $\frac{\pi}{12}$

D $-\frac{\pi}{6}$

iv. A class of 36 students is to be divided equally among the teacher teaching English, Dzongkha, Physics, Chemistry, Biology and Mathematics. In how many ways can these students be distributed among the subject groups for them to attend remedial classes?

A $\frac{(36)!}{6!}$

B $\frac{(36)!}{(6!)^6}$

C $\frac{(36)!}{6(6!)^6}$

D $\frac{(36)!}{6!(6!)^6}$

v. Evaluate: $\int_{-2}^{2} \log\left(\frac{1+x}{1-x}\right)dx$

A 0

B 1

C 2

D 3

vi. Which of the following differential equations are linear?

I $\frac{dy}{dx} = \frac{x^4 + 2xy + 1}{1 - x^2}$

II $\sin x \frac{dy}{dx} = y - \cos x$

III $y\frac{dy}{dx} = e^{\cos x} + \log x$

IV $\frac{d^2y}{dx^2} + 2xy - 5x = 0$

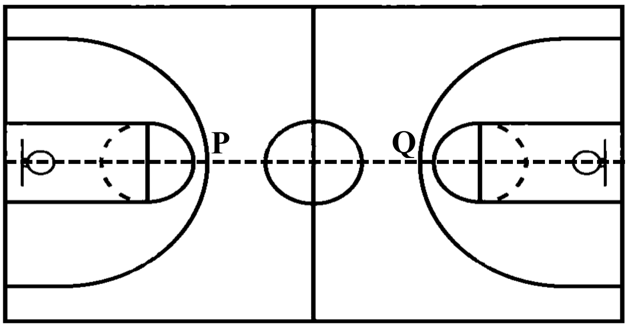
A I & III

B III & IV

C I, II & III

D I, II & IV

vii. If the equation of a 3-pointer curve of a basketball court is $9x^2 - 16y^2 = 144$, what is the horizontal distance between the points P and Q marked in the figure below?



A 6 units

B 8 units

C 9 units

D 16 units

viii. If $z_1 = 2 + 3i$ and $z_2 = 1 - 4i$, find the amplitude of $z_1 \times z_2$.

A -19.65^0

B -70.35^0

C 19.65^0

D 70.35^0

ix. The distance covered by a beetle crawling on the ground in time t seconds is given by $s = 2(t-2)^3 + 5t$. For the velocity to be minimum, what should be the time taken to cover the distance?

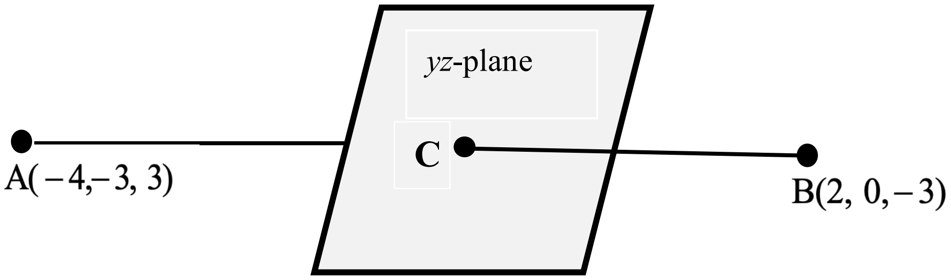
A $\frac{1}{6}$ sec

B $\frac{1}{2}$ sec

C 1 sec

D 2 sec

x. Find the coordinates of C.



A $(0,-1,-1)$

B $(-1, 0, -6)$

C $\left(0,\frac{-3}{2},\frac{-3}{2}\right)$

D $\left(-1,\frac{3}{2},\frac{-3}{2}\right)$

xi. In a fair, Karma and Sonam threw 6 identical darts at a target 10 *m* away. The following represents the probability associated between them:

Probability of Karma or Sonam not hitting the target is $\frac{1}{6}$

Probability of Karma and Sonam hitting the target is $\frac{1}{4}$

Probability of Karma not hitting the target is $\frac{1}{4}$

What is the relation between the events of hitting the target by both Karma and Sonam?

A Dependent and mutually exclusive.

B Independent and mutually exclusive.

C Dependent and not mutually exclusive.

D Independent and not mutually exclusive.

xii. What are the angles made by the normal to the plane $x-2y+2z-4=0$ with the axes?

A $Cos^{-1}\left(\frac{1}{3}\right), Cos^{-1}\left(\frac{2}{3}\right), Cos^{-1}\left(\frac{2}{3}\right)$

B $Cos^{-1}\left(\frac{1}{9}\right), Cos^{-1}\left(-\frac{2}{9}\right), Cos^{-1}\left(\frac{2}{9}\right)$

C $Cos^{-1}\left(\frac{1}{3}\right), Cos^{-1}\left(-\frac{2}{3}\right), Cos^{-1}\left(\frac{2}{3}\right)$

D $Cos^{-1}\left(\frac{1}{\sqrt{3}}\right), Cos^{-1}\left(\frac{2}{\sqrt{3}}\right), Cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$

xiii. Two judges were appointed to grade six areas in SUPW. Find x if the difference between the assigned ranks were as given below.

Class	**A**	**B**	**C**	**D**	**E**	**F**
Rank difference	−1	x	2.5	3.5	−2.5	−1.5

A 2.5

B 1

C −1

D −1.5

xiv. Athlete Mike Powell from the United States has a World record of 9 m in men's long jump. He initially accelerates for a few meters and then takes a jump once he gains momentum. If the locus of his jump is given by $y=9x-x^2$, what is the area under the curve and the horizontal ground?

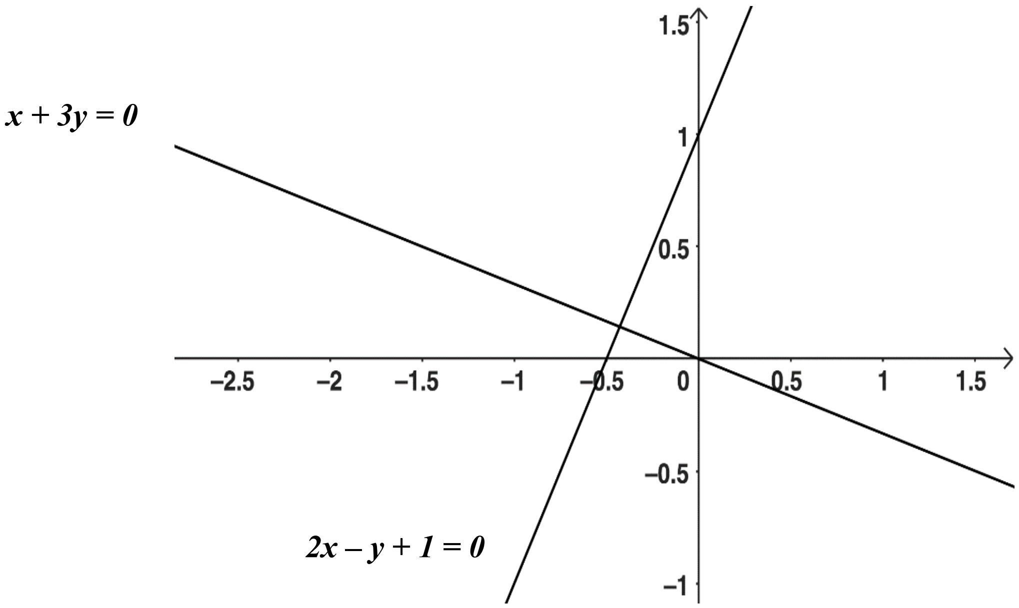
A 121.5 m^2

B 243.0 m^2

C 364.5 m^2

D 607.5 m^2

xv. Find the equation of a pair of lines passing through $(-2,1)$ and parallel to the given lines.



A $2x^2-5xy+3y^2+3x+16y-13=0$

B $2x^2+5xy-3y^2+3x+16y-13=0$

C $2x^2-7xy+3y^2+3x+14y+5=0$

D $2x^2+7xy-3y^2+3x+14y+5=0$

SECTION B [70 MARKS]
ATTEMPT **ANY TEN** QUESTIONS

Question 2

a) The table below shows the distribution in the consumption of different types of energy by % for cooking between rural and urban areas in Wangdue Dzongkhag as per Population and Housing Census of Bhutan 2017. Evaluate the coefficient of correlation of the main type of energy consumption between rural and urban areas. **[3]**

Area	Electricity	Kerosene	Firewood	Bio-Gas	LPG
Urban	98.6	0.3	0.3	1.8	92.4
Rural	95.8	1.3	12.9	1.3	79.8

b) Kinley, Dema and Laxmi have a total sum of Nu 284. Dema has one third of what Kinley has, and Laxmi has Nu 24 more than Dema. Using matrix method, calculate how much money each has. **[4]**

Question 3

a) Find the equation for the bisector of angles of the pair of lines perpendicular to the lines $-2x^2 + 5xy + y^2 = 0$. **[3]**

b) Kinley constructed an open tank with a square base of side x m and height h m to store $4\,m^3$ of water. He wants to paint the interior walls of the tank. What would be the minimum cost incurred if a litre of paint cost Nu 350 which can paint an area of $3\ m^2$? **[4]**

Question 4

a) To be awarded a Pass Certificate in the class XII examination, a student must pass in English and Dzongkha, and any other two subjects. A class XII Science student estimated his chance of passing in English as $\frac{4}{5}$, Physics as $\frac{5}{6}$, Chemistry as $\frac{2}{3}$ and Mathematics as $\frac{3}{4}$. What is the probability that he would be awarded a Pass Certificate on the condition that he received a pass mark in Dzongkha? **[3]**

b) Show that the equation $x^2 - 4xy + 4y^2 + 4x - 8y + 3 = 0$ represents a pair of parallel lines, and find the distance between them. **[4]**

Question 5

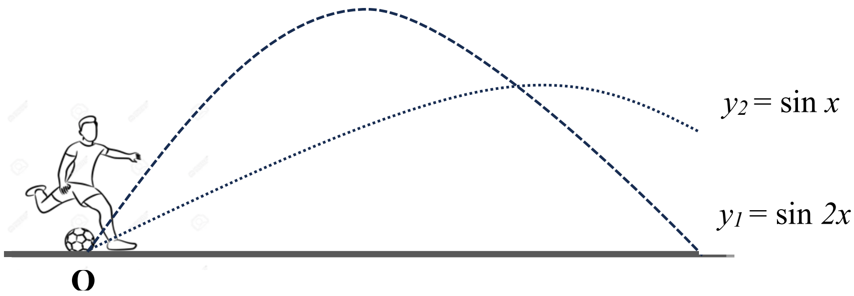
a) A school has 7 periods of 40 minutes each in a day. Commerce stream has 6 different subjects. In how many ways can we organize these subjects such that each subject is allocated at least one period in a day? **[3]**

b) Compute eccentricity, foci, equation of directrices, length of latus rectum and length of axes of conic section $8x^2 + 6y^2 = 96$. **[4]**

Question 6

a) If $A = \begin{bmatrix} 2 & x & -1 \\ 0 & 1 & y \\ 2 & -1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 3 & 1 \\ y & -1 & 1 \\ 2 & x & -1 \end{bmatrix}$ and $3A' + B = \begin{bmatrix} 10 & 3 & 7 \\ 2 & 2 & -2 \\ -1 & 2 & 8 \end{bmatrix}$. Find the value of x and y. **[3]**

b) Chencho kicked a ball twice and the path traced by the ball is shown in the figure below. Calculate the area enclosed between the two curves when O is the origin. **[4]**



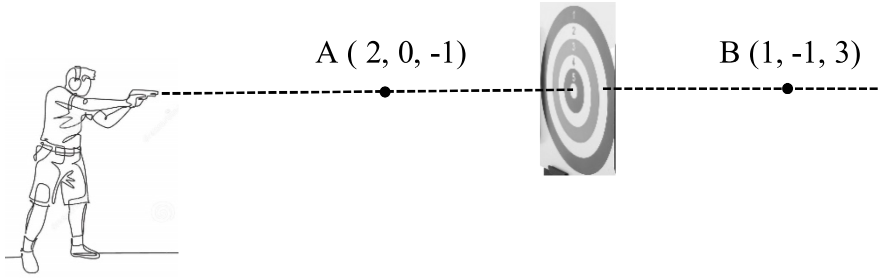
Question 7

a) The equation of two regression lines is $2y = 5x - 8$ and $3x - 4y + 12 = 0$. Evaluate the coefficient of correlation, regression coefficients b_{yx} and b_{xy}, mean of x and y and the value of x when $y = 11$. **[3]**

b) Verify if $y = -x\cos x + 2\sin x + c_1 x + c_2$ is the general solution for the differential equation $\frac{d^2y}{dx^2} = x\cos x$. Next, find the particular solution of the differential equation if $\frac{dy}{dx} = -1, y = 1$ when $x = 0$. **[4]**

Question 8

a) A man is shooting in an indoor firing range. The path of one of his bullets is perpendicular to the plane of a target as shown. If $P(2,-3,1)$ is any point on the target, find the equation of the plane. Also, find the equation of line AB. **[3]**



[4]

b) Illustrate that the locus of $\text{Re}(zi)^2 + 2x^2 = 4$, if $z = x + yi$ is a complex number.

Question 9

a) Prove that $9x^2 - 12xy\cos\theta + 4y^2 + 36\cos^2\theta - 36 = 0$, if $\cos^{-1}\left(\frac{x}{2}\right) + \cos^{-1}\left(\frac{y}{3}\right) = \theta$. **[3]**

b) A man going for a trek walks with a uniform velocity. Define a function which will describe the distance covered with respect to time. Graph your function and identify any two points on the x-axis. Rotate the area bounded by the graph and the line passing through the points about x-axis through four right angles and evaluate the volume of the shape so formed. **[4]**

Question 10

[3]

a) Convert $\frac{8-6i}{(1-i)^2}$ into polar form.

b) A house has a window in the form of a rectangle surmounted by a semicircle. If the perimeter of the window is 12 m, find the dimensions so as to allow maximum light into the room. **[4]**

Question 11

[3]

a) If $y = \sin(\tan^{-1} x)$, prove that $\frac{1}{3x}\frac{d^2y}{dx^2} = \frac{-1}{\left(1+x^2\right)^{\frac{5}{2}}}$.

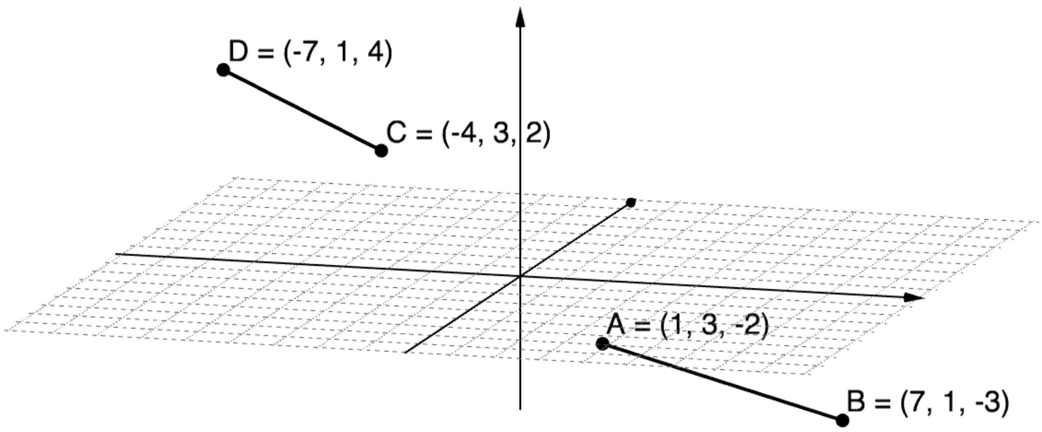
b) A bridge is built in the form of a parabolic arch. The highest point of the arch is 40 ft from the surface of water and has a span of 160 ft. Will a boat of 30 ft height be able to pass under the bridge if it is 24 ft from the center of the bridge? $(ft = \text{feet})$ **[4]**

Question 12

[3]

a) Integrate: $\int \frac{5x-1}{x^{2}(x-3)}\,dx$

b) Find the direction cosines of the line perpendicular to the lines AB and CD. **[4]**



Question 13

a) Kuenga has a 4 *m* ladder. He leans the ladder against the wall at a distance of 1 *m* from the building. Find the angle subtended by the ladder with the ground in radians. **[2]**

b) Wangmo draws two cards, one after another, from the deck of a well shuffled pack of cards. What is the probability of drawing:

i) either king or a queen in the first draw? **[1]**

ii) either a 10 or a club in the first draw? **[1]**

iii) king or queen of black in the first draw and queen or jack of red in the second draw if the first card drawn is replaced? **[1.5]**

iv) at least one diamond if the first card drawn is not replaced? **[1.5]**

FORMULAE

Strand A: Numbers and Operations

$a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$

$(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

If $ax^2 + bx + c = 0,\ x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

${}^nP_r = \dfrac{n!}{(n-r)!}$

${}^nC_r = \dfrac{n!}{r!(n-r)!}$

$C_{ij} = (-1)^{i+j} M_{ij}$

$A^{-1} = \dfrac{1}{|A|} adj.A$

$r = \sqrt{a^2 + b^2}$

$\tan\theta = \dfrac{b}{a} \Rightarrow \theta = \tan^{-1}\left|\dfrac{b}{a}\right|$

$z = r\left(\cos\theta + i\sin\theta\right)$

Strand B: Patterns and Algebra

$y = x^n,\ y' = nx^{n-1}$

$y = cf(x),\ y' = cf'(x)$

If $y = uv,\ \dfrac{dy}{dx} = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$

If $y = \dfrac{u}{v},\ \dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

$\dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx}$

$\int x^n dx = \dfrac{x^{n+1}}{n+1} + c$

$\int (ax+b)^n dx = \dfrac{(ax+b)^{n+1}}{a(n+1)} + c$

$\int uv dx = x\int v dx - \int\left\{\dfrac{du}{dx}\int v dx\right\}dx$

$\dfrac{dy}{dx} + Py = Q,\ I.F. = e^{\int P dx}$

$y(I.F.) = \int Q(I.F.)dx + c$

$1 + 2 + 3 + ... + (n-1) = \dfrac{1}{2}n(n-1)$

$1^2 + 2^2 + ... + (n-1)^2 = \dfrac{1}{6}n(n-1)(2n-1)$

$1^3 + 2^3 + 3^3 + ... + (n-1)^3 = \left[\dfrac{n(n-1)}{2}\right]^2$

$\int_a^b f(x)dx = \underset{h\to 0}{\text{Lim}}\, h\left[\sum_{r=0}^{n-1} f(a+rh)\right]$

$A = \int_a^b y dx,\ V = \pi\int_a^b y^2 dx$

Volume of Cone: $\dfrac{1}{3}\pi r^2 h$

Volume of Sphere: $\dfrac{4}{3}\pi r^3$

Volume of Cylinder: $\pi r^2 h$

Volume of Prism: $Base\ Area \times height$

S. Area of Cone: $\pi rl + \pi r^2$

S. Area of Sphere: $4\pi r^2$

S. Area of Cylinder: $2\pi rh + \pi r^2$

Area of sector: $\dfrac{1}{2}r^2\theta$

Strand C: Measurement

$\sin 3A = 3\sin A - 4\sin^3 A$

$\cos 3A = 4\cos^3 A - 3\cos A$

$\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

$\sin 2A = 2\sin A\cos A;$

$\cos 2A$

$= \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1;$

$\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A};$

$\sin^{-1} x \pm \sin^{-1} y = \sin^{-1}\left(x\sqrt{1-y^2} \pm y\sqrt{1-x^2}\right),$

If $x, y \geq 0$ & $x^2 + y^2 \leq 1;$

$\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}\left(xy \mp \sqrt{1-y^2} \cdot \sqrt{1-x^2}\right),$

If $x, y \geq 0$ & $x^2 + y^2 \leq 1.$

$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\dfrac{x+y}{1-xy}\right)$ *if* $xy < 1;$

$2\sin^{-1} x = \sin^{-1}\left(2x\sqrt{1-x^2}\right);$

$2\cos^{-1} x = \cos^{-1}\left(2x^2 - 1\right);$

$$2\tan^{-1} x = \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \sin^{-1}\left(\frac{2x}{1+x^2}\right) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right), \text{if } x^2 < 1$$

$3\sin^{-1} x = \sin^{-1}\left(3x - 4x^3\right)$

$3\cos^{-1} x = \cos^{-1}\left(4x^3 - 3x\right)$

$3\tan^{-1} x = \tan^{-1}\left(\dfrac{3x - x^3}{1 - 3x^2}\right)$

$\tan^{-1} x - \tan^{-1} y = \tan^{-1}\left(\dfrac{x-y}{1+xy}\right)$ if $xy > -1;$

Strand D: Geometry

$$\cos\theta = \pm\frac{a_1a_1 + b_1b_2 + c_1c_2}{\sqrt{a_1{}^2 + b_1{}^2 + c_1{}^2}\sqrt{a_2{}^2 + b_2{}^2 + c_2{}^2}}$$

If $a_1x + b_1y + c_1z = 0$ & $a_2x + b_2y + c_2z = 0,$

$$\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{z}{a_1b_2 - a_2b_1}$$

$$SP = ePM \Rightarrow \sqrt{(x-\alpha)^2 + (y-\beta)^2} = \left|\frac{ax + by + c}{\sqrt{a^2 + b^2}}\right|.$$

$\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$

Equation of angle bisectors: $\dfrac{x^2 - y^2}{a-b} = \dfrac{xy}{h}$

$l = \pm\dfrac{a}{\sqrt{a^2 + b^2 + c^2}}$

$m = \pm\dfrac{b}{\sqrt{a^2 + b^2 + c^2}}$

$n = \pm\dfrac{c}{\sqrt{a^2 + b^2 + c^2}}$

$\tan\theta = \left|\dfrac{2\sqrt{h^2 - ab}}{a+b}\right|$

Point of intersection:

$\left(\dfrac{hf - bg}{ab - h^2}, \dfrac{gh - af}{ab - h^2}\right)$

Strand E: Data Management and Probability

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n\sum x^2 - \left(\sum x\right)^2}\sqrt{n\sum y^2 - \left(\sum y\right)^2}}$$

$$r = \pm\sqrt{b_{yx} b_{xy}}$$

$$r = \frac{\sum\left(x - \bar{x}\right) - \sum\left(y - \bar{y}\right)}{\sqrt{\sum\left(x - \bar{x}\right)^2 - \sum\left(y - \bar{y}\right)^2}}$$

$$b_{yx} = r\frac{\sigma_y}{\sigma_x} = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - \left(\sum x\right)^2}$$

$$r = 1 - \frac{6\left(\sum D^2 + \text{correction factors}\right)}{n(n^2 - 1)}$$

$$b_{xy} = r\frac{\sigma_x}{\sigma_y} = \frac{n\sum xy - \sum x \sum y}{n\sum y^2 - \left(\sum y\right)^2}$$

Correction factor: $\frac{1}{12}\left(m^3 - m\right) + ...$

$$y - \bar{y} = b_{yx}\left(x - \bar{x}\right);\ x - \bar{x} = b_{xy}\left(y - \bar{y}\right)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(B / A) = \frac{P(A \cap B)}{P(A)},\ P(A) \neq 0.$$

$$P(A) + P(\bar{A}) = 1.$$

$$P(A / B) = \frac{P(A \cap B)}{P(B)},\ P(B) \neq 0.$$

Rough Work

Rough Work